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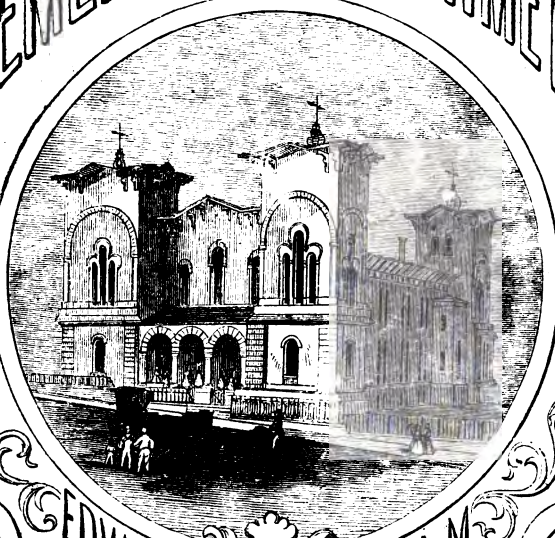

BY

EDWARD BROOKS, A.M.

PHILADELPHIA:

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DESIGNED FOR

PRIMARY SCHOOLS, AND PRIMARY CLASSES IN COMMON SCHOOLS GRADED SCHOOLS, MODEL SCHOOLS, ETC.

BY

EDWARD BROOKS, A. M.

PRINCIPAL AND PROFESSOR OF MATHEMATICS IN PENNSYLVANIA STATE NORMAL SCHOOL, AND AUTHOR OF NORMAL PRIMARY ARITHMETIC, NORMAL MENTAL ARITHMETIC, NORMAL WRITTEN ARITHMETIC, ELEMENTARY GEOMETRY, ETC.

"The highest science is the greatest simplicity."

PHILADELPHIA:

SOWER, POTTS & CO.,

530 MARKET ST., AND 523 MINOR ST.

1887.

OFFICE OF THE CONTROLLERS OF PUBLIC SCHOOLS,
FIRST DISTRICT OF PENNSYLVANIA.
PHILADELPHIA, March 29, 1869.

At a meeting of the Controllers of Public Schools, First District of Pennsylvania, held at the Controllers' Chamber, Tuesday, March 9, 1869, the following resolution was adopted:—

Resolved, That the Normal Series of Arithmetics, comprising "Brooks's Normal Primary Arithmetic," "Brooks's Normal Elementary Arithmetic," "Brooks's Normal Mental Arithmetic," and "Brooks's Normal Written Arithmetic," by Edward Brooks, Esq., be and the same are hereby placed on the list of books to be used in the Public Schools of this District.

From the minutes:

H. W. HALLIWELL, Sec'y.

MEARS & DUSENBERRY, ELECTROTYPERS. SHERMAN & CO., PRINTERS.

The Board of School Trustees for the STATE OF MARYLAND, recommend Brooks's Normal Arithmetics and Fewsmith's Grammars, for use in all the Public Schools of that state.

The Board of School Commissioners for the CITY OF BALTIMORE, have adopted Brooks's Normal Series and Fewsmith's Grammars, for exclusive use in the Public Schools of that city.

PREFACE.

THE object of this work is to furnish young pupils with an introductory course of *Written Arithmetic*. Realizing the necessity of such a course in connection with *Mental Exercises*, the author gave quite a large collection of written exercises in his Primary Mental Arithmetic. So popular was this feature that he was soon urged either to increase the amount of such matter or prepare another work treating exclusively of the elements of Written Arithmetic. Believing that the latter plan would be the more acceptable, and give more completeness to the arithmetical course, the present volume has been prepared.

This work will be found to possess at least five distinguishing features: 1st, Simplicity; 2d, Gradation; 3d, Practical Character of the Problems; 4th, Variety of Problems; 5th, Educational Character.

SIMPLICITY.—Great care has been taken to make the definitions, explanations, solutions, rules, etc. so simple that the youngest pupils can easily understand them. In doing this, however, the scientific character of the subject has not been sacrificed; for it should ever be remembered that *the highest science is the greatest simplicity;* and, conversely, *the greatest simplicity is the highest science.*

GRADATION.—The gradation of the work will be found one of its most distinctive and valuable features. A frequent criticism upon elementary works is their lack of gradation, their sudden transitions from the easy to the difficult, from the simple to the complex. To avoid this fault, great pains have been taken, and, it is believed, with success. For example, see the exercises in Addition, Subtraction, Multiplication, and Division, where the problems are arranged into *classes* and *cases* with respect to their length and difficulty. The same spirit of gradation will be found running through the whole work.

PRACTICAL PROBLEMS.—Arithmetics have been criticized for the abstract and unpractical character of their problems. To avoid this error, I have given a large number of practical problems, drawn from the actual events of life. Among these are Historical, Geographical, and Biographical problems; problems on the battles of the Revolution; farmers', merchants', etc., problems. Such problems will not only add interest to the study of arithmetic, but present much valuable information to the pupil.

VARIETY OF PROBLEMS.—"Variety is the spice of life," in the school-room as well as out of it. It is a great mistake to keep pupils upon Addition for several months, until they have thoroughly mastered it, then upon Subtraction for a corresponding length of time, and so on for the other fundamental operations. The better way is to give them a fair knowledge of Addition, then take them to Subtraction, and, after they are somewhat familiar with this, *give them exercises* combining Addition and Subtraction, and thus through the fundamental rules,

leaving each subject before the pupil wearies of it, and returning to it again and again, until it is thoroughly mastered. In this manner tiresome monotony is avoided, and the task of the learner rendered interesting and attractive. This is a feature of the present work which it is believed will commend it to intelligent instructors.

EDUCATIONAL CHARACTER.—This work, like the others of the same series, is characterized by an *educational spirit.* It is not a mere collection of problems and rules for the training of a pupil to labor like a machine. The spirit of analysis runs through it, making it *normal* in the broadest sense of the term. Its object is to teach pupils to *think* as well as to *work problems,*—to *develop mind* as well as the power of *computation.*

Cherishing the hope that it may aid teachers in their arduous labors, and become a favorite with the little girls and boys of our common schools, I now intrust it to the decision of a kind and appreciative public.

EDWARD BROOKS.

STATE NORMAL SCHOOL,
May 20, 1865.

SUGGESTIONS TO TEACHERS.

1. This book is designed to be put into the hands of young pupils soon after they begin the study of Primary Mental Arithmetic. When pupils can add and subtract orally with some facility, they will be prepared for this work.

2. The *Introduction* is not designed to be studied by the pupils, but indicates a course of *Oral Instruction* in the elements of arithmetic; and it is suggested that these exercises receive the attention which their importance demands. The pupils should have careful drill upon these before beginning the study of a written arithmetic; and such exercises will be found valuable during the entire course, especially upon commencing a new subject.

3. Pupils should solve the problems upon the slate at their seats, and also be required to work them out upon the blackboard, and explain them. In assigning problems at the board, the same problem may be given to the whole class, or each pupil may receive a different problem. Sometimes one method is preferred, and sometimes the other. The object should be thoroughness and accuracy, and at the same time variety and interest.

4. In many cases two forms of explanation have been given; one a full logical form, the other an abridged, mechanical one. The object of the first is to present the reasoning process in full; the object of the second, to give the steps in the method of operation. Though it is important that the pupil should understand the complete logical form, yet for the ordinary recitation, with young pupils, the abbreviated form may be preferred. It will economize time, and better secure that which is mainly aimed at in primary written arithmetic,—facility and accuracy of operation.

5. Where a pupil has difficulty with a problem owing to its being a little complicated, let the teacher lead him from one step to another, and so on to the end, by a judicious series of questions, leading him to analyze the problem, and thus unfold its complexity. This will be much better for the pupil than to pick up his slate and work the problem for him. By attention to these suggestions, and to such other points as will occur to the mind *of the intelligent teacher*, it is hoped that the progress of the pupil *will be rapid and thorough.*

INTRODUCTION.

ORAL EXERCISES.

THE following remarks and oral exercises are designed to illustrate the manner in which the elementary principles of numbers should be presented to the young pupil.

LESSON I.

NAMING NUMBERS.

Since our first ideas of numbers are derived from visible objects, the child's first lessons in the science of numbers should be given with such objects. These objects may consist of apples, nuts, books, pencils, grains of corn, or any thing the teacher may find convenient. The German schools for young children are generally supplied with small cubical blocks to be used in the first lessons on numbers. The *arithmetical frame* is the most convenient for general practice.

COUNTING.—The names of numbers are acquired simultaneously with the idea of numbers. Both of these are given in the process of *counting*. By counting we do not mean merely speaking the words *one*, *two*, *three*, etc., but that these words should be used in connection with objects, so that the pupil may know what the words mean. I have known pupils who could run off these words very glibly, even as far as *fifty*, without any definite idea of their meaning. Hence the pupil's first lesson in numbers should be in counting. The method suggested is as follows:—

The teacher, standing before the class, holding some object, as a book, in his hand, says, "What do I hold in my hand?" *Pupils:* "A book." *Teacher:* "How many books?" *Pupils:* "One book." The teacher, taking up another book, says, "How many books in *my hand now?*" *Pupils:* "Two books." And so on, until the

pupils can number any collection of objects which the teacher holds in his hand.

After this introduction, he will take the *arithmetical frame* and continue the exercises upon it. These exercises may be made lively by increasing or diminishing the number by several at the same time. Little *counting games*, with grains of corn, will also be found very interesting to young pupils.

Let exercises of this kind be continued until the class can count well. If the pupils can count when they enter school, this exercise need not be continued long.

LESSON II.

ADDITION AND SUBTRACTION.

After the pupils can readily number a collection of objects,—that is, have the *idea* of numbers and the *names* of numbers,—they should be taught to *unite* and *separate* them. The next thing in order, therefore, is *addition* and *subtraction*. Instruction in these processes should be given in accordance with the following principles.

1. *The first lessons in addition and subtraction should be given with visible objects.* This principle is founded upon the law of mental development, and is so evident that it need not be urged. Indeed, if the teacher neglects it, the pupil will adopt it himself, by adding with his fingers, etc.

2. *Addition and subtraction should be taught together in primary oral instruction.* This is evident, since the two ideas are logically related. Thus, as soon as the pupil learns that 2 and 3 are 5, he sees that 5 diminished by 2 equals 3, or 5 diminished by 3 is 2. Convenience also dictates the same method. In the primary schools of Germany, the two processes are combined, in the manner illustrated.

EXERCISE.—The order of exercises will now be given. The pupils should first *increase* and *diminish* by *one*, as far as 12, then by *two*, then by *three*, etc. The exercise would be somewhat as follows:—

Teacher takes one book in his hand, and asks, "How many books have I in my hand?"

PUPILS answer, "One book."

TEACHER, putting another book in his hand, asks, "How many books have I now?"

PUPILS answer, "Two books."

TEACHER: "How many, then, are one book and one book?"

PUPILS: "Two books.'

TEACHER "How many books have I in my hand now?"

PUPILS: "Two books."

TEACHER: "I will take one book away; now how many books remain?'

PUPILS: "One book."

TEACHER: "One book taken from two books, then, leaves how many books?"

PUPILS: "One book."

Let the teacher now take the *arithmetical frame*, and proceed in the same way, increasing 2, 3, 4, etc., up to 12 with *one*, and diminishing 3, 4, 5, 6, etc., up to 13 with *one*, each time reversing the addition. Then take one and increase it by *two*, obtaining three; then reverse the process and diminish three by *two*, and so on until 12 is increased by *two*, and 14 diminished by *two*. Proceed in the same way with 3, 4, etc., until the pupils can *add* and *subtract* by *ones*, *twos*, *threes*, etc., up to *twelves*.

LESSON III.

PRACTICAL EXERCISES.

The following exercises will be found valuable in teaching pupils to add and subtract with readiness and accuracy. Frequent drill upon such exercises is recommended.

First.—A valuable exercise to give the pupils readiness in adding and subtracting is the following: Let the teacher name two numbers, and require the pupils to give first their sum and then their difference: thus, teacher says, "5 and 2."

PUPILS: "5 and 2 are 7, and 2 from 5 leaves 3." After a little practice they may omit naming the numbers, and merely say, "the sum is 7, the difference is 3."

To vary this, the boys may give the sum, and the girls the difference, and *vice versâ;* or, if the class is all of one sex, a division may be made, one part giving the *sum*, and the other part the *difference.* In this and the following exercises care should be taken that small numbers be used at first, until the pupils attain the ability to use larger numbers with ease and readiness

Second.—Another valuable exercise consists in the teacher selecting some number, and then giving one part of this number himself, requiring the pupils to give the other part. For example, suppose 8 to be the number selected: the teacher says, "*five*," pupils answer, "*three*," teacher, "*two*," pupils, "*six*," etc. etc.

Third.—The following exercise will also be found valuable in imparting the art of adding and subtracting with readiness and accu-

racy. Let the teacher write the words *one*, *two*, *three*, etc., on the board, forming two columns as indicated in the margin. Call the first column additive, and the second subtractive. The teacher then with the pointer will indicate the number, the operation being indicated by the column. When he points to figures in the first column, the number which they indicate will be added, but when he points to any figure in the second, the number indicated will be subtracted from the result which the pupils have previously obtained. After the Arabic characters have been given, instead of writing the *words* in columns, the *figures* may be employed for the same purpose.

+	−
one	*one*
two	*two*
three	*three*
four	*four*
five	*five*
six	*six*
seven	*seven*
eight	*eight*
nine	*nine*

Fourth.—The pupils should also be required to add by *twos*, *threes*, etc., merely naming the results, as follows; 2, 4, 6, 8, etc., 3, 6, 9, etc., until the additions can be readily given.

It will be well to commence also with *one*, and count by *twos*, thus: 1, 3, 5, 7, etc.; also commence at 1, and count by *threes*, thus: 1, 4, 7, 10, etc.; also at 2, thus: 2, 5, 8, 11, etc., continuing the addition as far as it may be thought desirable.

Let the pupil be exercised in a similar manner in adding by *fours*, *fives*, etc., up to *twelves*. Such exercises should be continued day after day, in connection with the lessons which precede and follow this lesson, until great facility is acquired in the operations.

These exercises may be conducted sometimes in *concert* and sometimes *singly*. While one is adding alone, let the others keep careful watch for errors; a good degree of interest may thus be created, each pupil trying to obtain the largest sum before making a mistake.

It is evident that the teacher can give great variety to these exercises; and the author suggests that they will be found of very great utility.

LESSON IV.

WRITING NUMBERS.

The pupil should now be taught how to write numbers. In fact, the writing of numbers should be introduced very soon after the naming of numbers. The following exercises should, therefore, be combined with the exercises of Lesson III. The method suggested is as follows:—

The teacher, standing at the board, with some objects, as *two books in his hand, inquires*, "What do I hold in my hand?"

Answer:—"Books."

TEACHER: "How many books?"

PUPILS: "Two books."

TEACHER (writing *two books* upon the board) asks, "What have I written upon the board?"

PUPILS: "Two books."

TEACHER: "Are there two books on the board?"

PUPILS: "Yes, sir."

TEACHER: "If there are two books on the board, then what are these I hold in my hand?"

PUPILS: "Why, those are two books also."

TEACHER: "Well, if that is two books on the board, and these are two books in my hand, then they must both be the same."

PUPILS: "Oh, no, that on the board is the *words* two books, but what you have in your hand are the *things* two books."

Thus the important distinction between the *thing* and the *expression* of it is attained.

The teacher will now send the pupils to the board, and let them write the words *one*, *two*, etc., and have them solve problems in addition and subtraction with them. If the pupils cannot write (and I presume that will generally be the case at the time such exercises are appropriate), the teacher can illustrate by performing the written exercises himself.

The pupils will soon see the great labor of employing the written words, and will realize the necessity of an arithmetical written language different from that which is used in ordinary writing. They are now prepared for the Arabic characters; and the manner in which these should be introduced will now be given.

CHARACTERS.—The pupils being now prepared for the Arabic characters, let the teacher give first in their order the *nine digits*. They should be exercised in naming and writing these until they are familiar with them and can make them with considerable ease and neatness. They should then be required to solve problems with them in addition and subtraction, the teacher giving no problem at present which involves a number greater than nine.

COMBINATIONS.—When the class are familiar with these characters, they are to be taught to combine them to express the larger numbers. There are two methods of doing this, quite different in principle and form. We present both.

First Method.—By this method we give the *combined characters, without explaining* the principle of the combination. Thus, we *teach that 10* represents *ten*, 11, *eleven*, 12, twelve, etc., without any

reference to tens and units. This method is not quite so philosophical as the second method, but will be found preferable in practice with young learners in oral instruction. When pupils study written arithmetic from the book, I would use the other method.

We would give these expressions as far as *twenty*, and then drill the pupils in reading and writing them until they are quite familiar with them. We would next give the expressions from *twenty* to *thirty*, and drill in like manner, and thus continue as far as *one hundred.*

After the pupils are familiar with this method of writing numbers as far as 100, the teacher may then show them the principle of the combination, that the figure in the first place represents *units*, in the second place *tens*, etc. When this is understood, we would require the class to analyze these expressions, as follows:—

Problem.—Analyze 25 (*twenty-five*).

Analysis.—In 25, the 5 represents 5 *units*, and the 2 represents 2 *tens.*

The class should also be drilled upon questions like the following:—How many units in 2 *tens?* In 3 *tens?* etc. How many tens in 20 (*twenty*)? etc. How many units and tens in 24 (*twenty-four*)? They should also be required to solve little problems in addition and subtraction with these characters.

Second Method.—The other method commences by explaining the principle of the combination; that is, that 10 represents 1 *ten;* 11, 1 *ten* and 1 *unit;* 12, 1 *ten* and 2 *units*, etc., afterward showing that 11 (1 *ten* and 1 *unit*) is the same as *eleven*, etc.

This may be done by making ten marks upon the board, and then commencing a second row with one mark; these will be represented by 11 (1 *ten* and 1 *unit*); then have the pupils count them, and they will see that 11 (1 *ten* and 1 *unit*) stands for *eleven.* The same may be done with 12, 13, etc.

The pupil should be drilled in reading and writing numbers, until he is entirely familiar with the subject. Haste here is "bad speed." A thorough knowledge of Notation and Numeration will dispel the usual difficulties of Addition, Subtraction, Multiplication, and Division.

LESSON V.

MULTIPLICATION AND DIVISION.

After the pupil has become quite familiar with the elementary *processes of Addition* and Subtraction, he is prepared to take up *Multiplication and Division.* The first instruction in these processes

should be given by oral exercises, and in accordance with the following principles:

1. *Multiplication should be presented as a special case of Addition.* Thus, the pupil should be taught that two 2's are 4, since $2+2=4$, or that *three* times 2 are 6, since 2 taken three times, or $2+2+2$, equals 6, and so on for the other products. The pupil will then understand the nature of the subject.

2. *Division should be taught as reverse multiplication.* Thus, it should be shown that 6 contains 3 *two* times, since *two* times 3 are 6, and so on for other quotients. In this way the quotients are immediately derived from the products. Division may be taught as *concise subtraction*, but the process of *reverse multiplication* is more convenient and simple. When thus taught, the pupil will not need to commit a distinct Division Table.

3. *The pupil should be taught to construct the Multiplication Table.* The pupil should not be required to commit a Multiplication Table without knowing how it was obtained, or the use of it. He should first be taught to derive the products for himself, by addition, and then be required to commit them, to avoid the labor of obtaining them every time he wishes to use them. In this way he will study them with more interest, and learn them with greater ease.

4. *Multiplication and Division should be taught simultaneously, or at least very nearly so.* As soon as the pupil learns that 2 times 3 are 6, he is able to see that 6 equals two 3's, or that 6 contains 3 *two times;* and the same is true for the other products. Division, therefore, should not be deferred until the whole Multiplication Table is learned, as has generally been the practice, but should be early introduced and studied in connection with Multiplication.

We now present the following exercise, which is designed to suggest the manner in which the first principles of multiplication and division may be taught.

EXERCISE.

TEACHER (making two marks on the board, as is indicated in the margin) asks, "How many marks have I made?"

PUPILS: "Two marks."

TEACHER (making two marks under the former two, as is indicated in the margin, inquires): "How many times have I made two marks?"

PUPILS: "Two times."

TEACHER: "How many marks are there in all?"

PUPILS: "Four marks."

TEACHER: "How many, then, are *two* times two marks?"

PUPILS: "Two times two marks are four marks."

TEACHER (leaving the four marks upon the board, asks): "How many marks are there on the board?"

PUPILS: "Four marks."

TEACHER: "Are they arranged in *twos*, or *threes*?"

PUPILS: "In twos."

TEACHER: "How many *twos* are there in these four marks?"

PUPILS: "Two twos."

TEACHER: "Four, then, contains two how many times?"

PUPILS: "Two times."

| | |	The teacher will now write six marks upon the board, as in the margin, and ask, "How many times have I written three marks?" "How many are there in all?" "How many, then, are two times three?" "Are these 6 marks arranged in *twos*, *threes*, or *fours*?" "How many times three marks are there?" "How many *threes*, then, are there in six?" "Six, then, contains 3 how many times?"

These, or similar exercises, should be continued up to two times 12, each time reversing the process, and obtaining a quotient. Then proceed in the same way with *three* times, *four* times, etc., on to *twelve* times. In this manner the pupils may be led to *obtain*, and then *commit*, the products and quotients, usually given in tables, which are always learned with so much hesitation and hard study.

In practice, it will be well to obtain all the *products* of "*two times*," before deriving the *quotients*. Questions similar to those in the Primary Arithmetic, p. 32, may also be given. The pupil should also be taught to write the table of products upon the slate or board, thus:

$1 \times 2 = 2$	$4 \times 2 = 8$
$2 \times 2 = 4$	$5 \times 2 = 10$
$3 \times 2 = 6$	etc.

Pupils generally have considerable difficulty in committing the Multiplication Table; the teacher can lessen the labor in several ways. 1st. By having the pupils make it for themselves, and write it on the slate or blackboard. 2d. By concert recitation. 3d. By *singing* the table to some appropriate tune. 4th. Reciting by the old *method of "going up,"* or "*trapping*."

To make pupils rapid and accurate in the mechanical processes of

addition, subtraction, multiplication, and division, the following exercise is practised by some teachers, with excellent results. Let the teacher write four columns of figures on the blackboard, as is represented in the margin, the first column being additive, the next subtractive, etc., as is indicated by the symbols placed above them. The teacher, with the pointer, will point out certain figures, the corresponding numbers being added, subtracted, multiplied, or divided, as is indicated by the symbol at the head of the column. Care, of course, must be taken not to require a division by a number that is not exactly contained. This exercise may be continued for many recitations, in connection with the following lessons, with great advantage to the pupils.

(+)	(−)	(×)	(÷)
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

LESSON VI.

TERMS AND PRINCIPLES.

The following exercises are designed to suggest the manner of giving the *terms* of the fundamental rules, and also of deriving some of the principles of each.

ADDITION AND SUBTRACTION.

TEACHER: "What have I in my hand?"

PUPILS: "Two books."

TEACHER: "What is the difference between the *two* and the books?"

PUPILS: "The *books* are the *things*, and the *two* tells *how many* things."

TEACHER: "The *two* denotes the *number* of books. What, then, is a *number?*"

PUPILS: "Why, it is the *how many* of any thing."

TEACHER: "Very well; remember, also, that a single thing, or *one* of a collection, is called a *unit*."

TEACHER: "When I say two apples, what 2 do I mean?"

PUPILS: "Two *apples*."

TEACHER: "When I say *two*, what do I mean?"

PUPILS: "We do not know."

TEACHER: "What 2 may I mean? *any* two?"

PUPILS: "Yes, sir; any two you choose."

TEACHER: "You see a difference, then, between two and two

books. Very well; I will give the name which denotes this difference. When I say 2, 3, etc., without telling what 2 or 3, it is called an *abstract number*; but when I give the name of the objects with the number, it is called a *concrete number.*"

Tell which of the following numbers are abstract, and which concrete:—

2 cows—three—four—4 books—7 hens—eight—5—4—10 pigs—8 geese—7—6—11—14 horses.

TEACHER: How many are 3 and 5?

TEACHER: When we unite two numbers into one, in this way, the result is called the *sum*, and the process is called ADDITION.

TEACHER: What is the sum of 2 and 3? 4 and 6? 7 and 8?

TEACHER: What is the sum of 3 cows and 5 turnips?

TEACHER: Why can you not add them?

TEACHER: If they were all the same, could you add them?

TEACHER: Numbers which express the same kind of objects are *similar concrete* numbers, and those which denote different objects are *dissimilar concrete* numbers.

TEACHER: What kind of numbers can be added, then, and what kind cannot be added?

How many remain when we take 3 apples from 5 apples?

The process of taking one number from another is called SUBTRACTION.

The number which is taken away is called the SUBTRAHEND, the number from which it is taken is called the MINUEND, and the result is called the DIFFERENCE, or REMAINDER.

If you subtract 4 from 9, which is the *minuend*, which the *subtrahend*, and which the *remainder?*

If you add the remainder and subtrahend together, will it produce the minuend?

If you subtract the difference from the minuend, what will it equal?

Can you subtract 3 apples from 5 potatoes?

Why can you not subtract them?

Are these similar or dissimilar concrete numbers?

If they were similar, could they be subtracted?

What kind of numbers, then, can be subtracted, and what kind cannot?

PRINCIPLES OF MULTIPLICATION AND DIVISION.

1. When we find the result of a number taken any number *of times, the process is* called *multiplication.*

2. The number taken a certain number of times is called the *multiplicand.*

3. The number which denotes how many times the multiplicand is taken is called the *multiplier.*

4. The result obtained is called the *product.* Each of these three is called a *term.*

5. What is the product of 8 apples multiplied by 4?

6. In this problem, which is the multiplicand, which the multiplier, which the product?

7. When we take 8 apples 4 times, is the result apples, or something else?

8. Can the product be any thing else than apples?

9. The product, then, is of the same denomination as what term?

10. Can we take 8 apples 4 *peaches* times, or simply 4 times?

11. Is 4 an abstract or a concrete number? What kind of a number, then, must the multiplier be?

12. When we find how many times one number is contained in another, the process is called *division.*

13. The number which contains the other is called the *dividend,* the number contained is called the *divisor,* and the number denoting how many times the divisor is contained is called the *quotient.*

14. If we divide 8 apples by 2 apples, is the result apples? If not, what is it?

15. Are 2 apples contained in 8 apples 4 peaches times, or 4 apples times, or simply 4 times?

16. What kind of a number is 4, and what kind of a number, then, must the *quotient* always be?

17. How many times 2 equal 8 apples? Is 2, or 2 pears, contained any number of times in 8 apples?

18. What 2 are contained any number of times in 8 apples?

19. The *divisor,* then, must be of the same denomination as what term?

LESSON VII.

TABLE OF FUNDAMENTAL RULES.

We now present the *tables of the four fundamental rules,* for such teachers as wish to use them. The author suggests that the elementary *sums* and *differences* are better taught by the exercises which have been already suggested than by the study of these tables. The Multiplication Table, however, must be thoroughly committed, and then the elementary quotients may be derived from *these products,* or by the study of the Division Table.

ADDITION TABLE.

2 and	3 and	4 and	5 and
0 are 2	0 are 3	0 are 4	0 are 5
1 " 3	1 " 4	1 " 5	1 " 6
2 " 4	2 " 5	2 " 6	2 " 7
3 " 5	3 " 6	3 " 7	3 " 8
4 " 6	4 " 7	4 " 8	4 " 9
5 " 7	5 " 8	5 " 9	5 " 10
6 " 8	6 " 9	6 " 10	6 " 11
7 " 9	7 " 10	7 " 11	7 " 12
8 " 10	8 " 11	8 " 12	8 " 13
9 " 11	9 " 12	9 " 13	9 " 14
10 " 12	10 " 13	10 " 14	10 " 15
11 " 13	11 " 14	11 " 15	11 " 16
12 " 14	12 " 15	12 " 16	12 " 17

6 and	7 and	8 and	9 and
0 are 6	0 are 7	0 are 8	0 are 9
1 " 7	1 " 8	1 " 9	1 " 10
2 " 8	2 " 9	2 " 10	2 " 11
3 " 9	3 " 10	3 " 11	3 " 12
4 " 10	4 " 11	4 " 12	4 " 13
5 " 11	5 " 12	5 " 13	5 " 14
6 " 12	6 " 13	6 " 14	6 " 15
7 " 13	7 " 14	7 " 15	7 " 16
8 " 14	8 " 15	8 " 16	8 " 17
9 " 15	9 " 16	9 " 17	9 " 18
10 " 16	10 " 17	10 " 18	10 " 19
11 " 17	11 " 18	11 " 19	11 " 20
12 " 18	12 " 19	12 " 20	12 " 21

10 and	11 and	12 and	13 and
0 are 10	0 are 11	0 are 12	0 are 13
1 " 11	1 " 12	1 " 13	1 " 14
2 " 12	2 " 13	2 " 14	2 " 15
3 " 13	3 " 14	3 " 15	3 " 16
4 " 14	4 " 15	4 " 16	4 " 17
5 " 15	5 " 16	5 " 17	5 " 18
6 " 16	6 " 17	6 " 18	6 " 19
7 " 17	7 " 18	7 " 19	7 " 20
8 " 18	8 " 19	8 " 20	8 " 21
9 " 19	9 " 20	9 " 21	9 " 22
10 " 20	10 " 21	10 " 22	10 " 23
11 " 21	11 " 22	11 " 23	11 " 24
12 " 22	12 " 23	12 " 24	12 " 25

SUBTRACTION TABLE.

1 from			2 from			3 from			4 from		
1	leaves	0	2	leaves	0	3	leaves	0	4	leaves	0
2	"	1	3	"	1	4	"	1	5	"	1
3	"	2	4	"	2	5	"	2	6	"	2
4	"	3	5	"	3	6	"	3	7	"	3
5	"	4	6	"	4	7	"	4	8	"	4
6	"	5	7	"	5	8	"	5	9	"	5
7	"	6	8	"	6	9	"	6	10	"	6
8	"	7	9	"	7	10	"	7	11	"	7
9	"	8	10	"	8	11	"	8	12	"	8
10	"	9	11	"	9	12	"	9	13	"	9
11	"	10	12	"	10	13	"	10	14	"	10
12	"	11	13	"	11	14	"	11	15	"	11
13	"	12	14	"	12	15	"	12	16	"	12

5 from			6 from			7 from			8 from		
5	leaves	0	6	leaves	0	7	leaves	0	8	leaves	0
6	"	1	7	"	1	8	"	1	9	"	1
7	"	2	8	"	2	9	"	2	10	"	2
8	"	3	9	"	3	10	"	3	11	"	3
9	"	4	10	"	4	11	"	4	12	"	4
10	"	5	11	"	5	12	"	5	13	"	5
11	"	6	12	"	6	13	"	6	14	"	6
12	"	7	13	"	7	14	"	7	15	"	7
13	"	8	14	"	8	15	"	8	16	"	8
14	"	9	15	"	9	16	"	9	17	"	9
15	"	10	16	"	10	17	"	10	18	"	10
16	"	11	17	"	11	18	"	11	19	"	11
17	"	12	18	"	12	19	"	12	20	"	12

9 from			10 from			11 from			12 from		
9	leaves	0	10	leaves	0	11	leaves	0	12	leaves	0
10	"	1	11	"	1	12	"	1	13	"	1
11	"	2	12	"	2	13	"	2	14	"	2
12	"	3	13	"	3	14	"	3	15	"	3
13	"	4	14	"	4	15	"	4	16	"	4
14	"	5	15	"	5	16	"	5	17	"	5
15	"	6	16	"	6	17	"	6	18	"	6
16	"	7	17	"	7	18	"	7	19	"	7
17	"	8	18	"	8	19	"	8	20	"	8
18	"	9	19	"	9	20	"	9	21	"	9
19	"	10	20	"	10	21	"	10	22	"	10
20	"	11	21	"	11	22	"	11	23	"	11
21	"	12	22	"	12	23	"	12	24	"	12

MULTIPLICATION TABLE.

Once			2 times			3 times			4 times		
1	is	1	1	are	2	1	are	3	1	are	4
2	"	2	2	"	4	2	"	6	2	"	8
3	"	3	3	"	6	3	"	9	3	"	12
4	"	4	4	"	8	4	"	12	4	"	16
5	"	5	5	"	10	5	"	15	5	"	20
6	"	6	6	"	12	6	"	18	6	"	24
7	"	7	7	"	14	7	"	21	7	"	28
8	"	8	8	"	16	8	"	24	8	"	32
9	"	9	9	"	18	9	"	27	9	"	36
10	"	10	10	"	20	10	"	30	10	"	40
11	"	11	11	"	22	11	"	33	11	"	44
12	"	12	12	"	24	12	"	36	12	"	48

5 times			6 times			7 times			8 times		
1	are	5	1	are	6	1	are	7	1	are	8
2	"	10	2	"	12	2	"	14	2	"	16
3	"	15	3	"	18	3	"	21	3	"	24
4	"	20	4	"	24	4	"	28	4	"	32
5	"	25	5	"	30	5	"	35	5	"	40
6	"	30	6	"	36	6	"	42	6	"	48
7	"	35	7	"	42	7	"	49	7	"	56
8	"	40	8	"	48	8	"	56	8	"	64
9	"	45	9	"	54	9	"	63	9	"	72
10	"	50	10	"	60	10	"	70	10	"	80
11	"	55	11	"	66	11	"	77	11	"	88
12	"	60	12	"	72	12	"	84	12	"	96

9 times			10 times			11 times			12 times		
1	are	9	1	are	10	1	are	11	1	are	12
2	"	18	2	"	20	2	"	22	2	"	24
3	"	27	3	"	30	3	"	33	3	"	36
4	"	36	4	"	40	4	"	44	4	"	48
5	"	45	5	"	50	5	"	55	5	"	60
6	"	54	6	"	60	6	"	66	6	"	72
7	"	63	7	"	70	7	"	77	7	"	84
8	"	72	8	"	80	8	"	88	8	"	96
9	"	81	9	"	90	9	"	99	9	"	108
10	"	90	10	"	100	10	"	110	10	"	120
11	"	99	11	"	110	11	"	121	11	"	132
12	"	108	12	"	120	12	"	132	12	"	144

DIVISION TABLE.

1 in		2 in		3 in		4 in	
1	1 time	2	1 time	3	1 time	4	1 time
2	2 times	4	2 times	6	2 times	8	2 times
3	3 "	6	3 "	9	3 "	12	3 "
4	4 "	8	4 "	12	4 "	16	4 "
5	5 "	10	5 "	15	5 "	20	5 "
6	6 "	12	6 "	18	6 "	24	6 "
7	7 "	14	7 "	21	7 "	28	7 "
8	8 "	16	8 "	24	8 "	32	8 "
9	9 "	18	9 "	27	9 "	36	9 "
10	10 "	20	10 "	30	10 "	40	10 "
11	11 "	22	11 "	33	11 "	44	11 "
12	12 "	24	12 "	36	12 "	48	12 "

5 in		6 in		7 in		8 in	
5	1 time	6	1 time	7	1 time	8	1 time
10	2 times	12	2 times	14	2 times	16	2 times
15	3 "	18	3 "	21	3 "	24	3 "
20	4 "	24	4 "	28	4 "	32	4 "
25	5 "	30	5 "	35	5 "	40	5 "
30	6 "	36	6 "	42	6 "	48	6 "
35	7 "	42	7 "	49	7 "	56	7 "
40	8 "	48	8 "	56	8 "	64	8 "
45	9 "	54	9 "	63	9 "	72	9 "
50	10 "	60	10 "	70	10 "	80	10 "
55	11 "	66	11 "	77	11 "	88	11 "
60	12 "	72	12 "	84	12 "	96	12 "

9 in		10 in		11 in		12 in	
9	1 time	10	1 time	11	1 time	12	1 time
18	2 times	20	2 times	22	2 times	24	2 times
27	3 "	30	3 "	33	3 "	36	3 "
36	4 "	40	4 "	44	4 "	48	4 "
45	5 "	50	5 "	55	5 "	60	5 "
54	6 "	60	6 "	66	6 "	72	6 "
63	7 "	70	7 "	77	7 "	84	7 "
72	8 "	80	8 "	88	8 "	96	8 "
81	9 "	90	9 "	99	9 "	108	9 "
90	10 "	100	10 "	110	10 "	120	10 "
99	11 "	110	11 "	121	11 "	132	11 "
108	12 "	120	12 "	132	12 "	144	12 "

THE

NORMAL

ELEMENTARY ARITHMETIC.

SECTION I.

1. A ***Unit*** is a single thing, as a *book, pen, apple.*

2. A ***Number*** is one or more things, as *two* books, *three* pens.

3. ***Arithmetic*** is the science of numbers and the art of using them.

4. ***Mental Arithmetic*** is the solving of problems without the aid of written characters.

5. ***Written Arithmetic*** is the solving of problems with written characters.

6. In studying arithmetic we first learn to *name* numbers, and then learn to *write* them.

NUMERATION.

7. ***Numeration*** is the art of naming numbers, and of reading them when they are written.

8. We will give the names of some of the numbers up to one hundred.

one;	six;
two;	seven;
three;	eight;
four;	nine;
five;	ten;

eleven, or one and ten;
twelve, or two and ten;
thirteen, or three and ten;
fourteen, or four and ten;
fifteen, or five and ten;
sixteen, or six and ten;
seventeen, or seven and ten;
eighteen, or eight and ten;
nineteen, or nine and ten;
twenty, or two tens;
twenty-one, or two tens and one;
twenty-two, or two tens and two;
thirty, or three tens;
thirty-one, or three tens and one;
forty, or four tens;
forty-one, or four tens and one;
fifty, or five tens;
fifty-one, or five tens and one;
sixty, or six tens;
sixty-one, or six tens and one;
seventy, or seven tens;
etc., etc.

NOTE.—The pupil should be drilled upon these equivalent forms of naming numbers, as a preparation for Notation. The teacher or pupil may fill out the omissions.

ARABIC NOTATION.

9. ***Notation*** is the art of writing numbers.

10. ***Figures.***—In writing numbers we use the following ten characters, called *figures*.

1	expresses	one.	6	expresses	six.
2	"	two.	7	"	seven.
3	"	three.	8	"	eight.
4	"	four.	9	"	nine.
5	"	five.	0	"	naught.

11. ***Combination.***—By these figures and their combinations all numbers can be expressed.

The method of combining them is as follows:—

1st. *A figure standing alone expresses units or ones.*

2d. *When two figures are together, the one in the first place at the right expresses units, the one in the second place expresses tens.*

3d. *A figure in the third place expresses hundreds, in the fourth place thousands*, etc.

12. Thus, in 25, the 2 expresses 2 *tens*, and the 5 expresses 5 *units*. We will illustrate this by the following *table*.

10 is one ten.	11 is 1 ten and one.
20 " two tens.	12 " 1 ten and two.
30 " three tens.	23 " 2 tens and three
40 " four tens.	34 " 3 tens and four.
50 " five tens.	47 " 4 tens and seven.
60 " six tens.	58 " 5 tens and eight.
70 " seven tens.	65 " 6 tens and five.
80 " eight tens.	79 " 7 tens and nine.
90 " nine tens.	86 " 8 tens and six.
100 " one hundred.	105 " 1 hundred and five.

13. The pupils will now write and read the following numbers:

13	24	33	41	57	74
15	26	36	43	60	76
17	27	38	45	68	82
19	29	39	50	63	95
21	30	40	52	70	126

14. The pupils will now learn the names of the first twelve places, as represented in the following

NUMERATION TABLE.

NAMES.	Hundred-billions	Ten-billions.	Billions.	Hundred-millions	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
	4	4	4	4	4	4	4	4	4	4	4	4
PLACES.	12th,	11th,	10th,	9th,	8th,	7th,	6th,	5th,	4th,	3d,	2d,	1st,
PERIODS.	4th.			3d			2d.			1st.		

15. The pupil should now be drilled upon questions similar to the following.

Required the names of the following places:

1. First.	4. Second.	7. Eighth.
2. Third.	5. Fifth.	8. Seventh.
3. Fourth.	6. Sixth.	9. Ninth.

Required the places of the following:

1. Tens.	4. Thousands.
2. Hundreds.	5. Millions.
3. Ten-thousands.	6. Hundred-thousands.

16. ***Periods.***—For convenience in writing and reading numbers, we arrange the figures in periods of three places each, as shown by the table.

The first three places make the *first* or *units, period,* the second three places make the *second* or *thousands, period,* etc.

Required the period and place of the following:

1. Hundreds.	5. Tens.
2. Thousands.	6. Ten-millions.
3. Millions.	7. Hundred-thousands.
4. Ten-thousands.	8. Hundred-millions.

17. The combination of figures to express a number forms a *numerical word.* Thus, 25 is the numerical word which means the same as *twenty-five.* These numerical words may be analyzed.

PROBLEMS.

1. Analyze the numerical word 324.

ANALYSIS.—The 4 represents *four units,* the 2 represents *two tens,* the 3 represents *three hundreds:* hence the numerical word is *three hundred and twenty-four.*

Analyze the following:

2. 426	4. 652	6. 853	8. 395	10. 1234	12. 43762
3. 357	5. 785	7. 687	9. 785	11. 5678	13. 85967

18. We will now give some exercises in Numeration and Notation.

RULE FOR NUMERATION.

I. *Begin at the right hand, and separate the numerical word into periods of three figures each.*

II. *Then begin at the left hand, and read each period as if it stood alone, giving the name of each period except the last.*

1. What number is expressed by 3254789?

SOLUTION.—We separate the numerical word into periods of three figures each, as in the margin. The third period is 3 millions, the second period is 254 thousands, the first is 789; hence the number is 3 millions, 254 thousands, 789.

OPERATION.
3,254,789

Read the following:

2.	2384	6.	64327	10.	6321456
3.	7428	7.	52105	11.	78535217
4.	6321	8.	43246	12.	852106721
5.	8357	9.	785625	13.	12345678935

19. Having learned to read numerical words, the pupils are now prepared to write them. From the principles we have given we derive the following

RULE FOR NOTATION.

Begin at the left, and write each figure in order towards the right, giving each figure its proper place, and filling the vacant places with ciphers.

1. Write the number four thousand three hundred and seven.

SOLUTION.—We write the 4 in thousands' place, the 3 in hundreds' place, the 7 in units' place, and, since *there are no tens*, we write a naught in tens' place.

OPERATION.
4307

Write the following in figures:

2. Three thousand and seventy-five. Ans. 3075

3. Five thousand six hundred and fifty.

4. Seven thousand eight hundred and four.

5. Twenty-three thousand four hundred and ninety.

6. Twenty-five thousand three hundred and seven.

7. Two hundred and six thousand four hundred and six.

8. Four hundred and eighty-six thousand nine hundred and eight.

9. Seven hundred and forty-three thousand four hundred and ninety.

10. Two millions, three hundred thousand four hundred and eighty.

11. Four millions, five hundred and six thousand and twenty-five.

12. Six billions, six millions, six thousands, six hundred and six.

REMARK.—Pupils should be drilled in exercises like those given, until they can read and write numbers readily.

ROMAN NOTATION.

The Roman Method of Notation employs seven letters of the Roman alphabet. Thus, I represents *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* M, *one thousand.*

To express other numbers these characters are combined according to the following principles:—

1. *Every time a letter is repeated its value is repeated.*

2. *When a letter is placed before one of greater value, the* DIFFERENCE *of their values is the number represented.*

3. *When a letter is placed after one of a greater value, the* SUM *of their values is the number represented.*

4. *A dash placed over a letter increases its value a thousand fold. Thus,* $\overline{\text{VII}}$ *denotes seven thousand.*

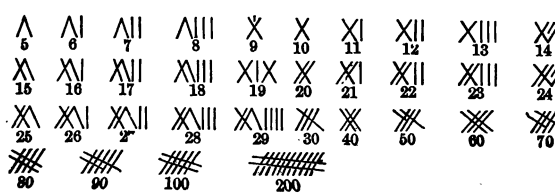
ROMAN TABLE.

I	One.	XXX	Thirty.
II	Two.	XL	Forty.
III	Three.	L	Fifty.
IV	Four.	LX	Sixty.
V	Five.	LXX	Seventy.
VI	Six.	XC	Ninety.
VII	Seven.	C	One hundred.
VIII	Eight.	CC	Two hundred.
IX	Nine.	D	Five hundred.
X	Ten.	DC	Six hundred.
XI	Eleven.	DCCCC	Nine hundred.
XIV	Fourteen.	M	One thousand.
XV	Fifteen.	MM	Two thousand.
XIX	Nineteen.	MCLX	One thousand one hun-
XX	Twenty	MDCCCLIX	1859. [dred and sixty

The Roman Method is named after the Romans, who invented and used it. It is now employed to denote the chapters and sections of books, pages of preface and introduction, and in other places for prominence and distinction.

LUMBERMEN'S NOTATION.

Lumbermen in marking lumber employ a modification of the Roman Method of Notation. The first four characters are like the Roman; the others are as follows:



SECTION II.

ADDITION.

20. ***Addition*** is the process of finding the *sum* of two or more numbers.

21. The ***Sum*** is a number which contains as many units as the numbers added.

22. The ***sign of Addition*** is +, and is read *plus*. The ***sign of Equality*** is =, and is read *equals*, or *equal to*. Thus, 4 + 5 = 9, is read 4 *plus* 5 *equals* 9.

CASE I.

23. To add when the sum of a column is not more than nine of that column.

24. *CLASS I.—Problems of one column.*

1. What is the sum of 2, 3, and 4?

SOLUTION.—We write the numbers one under another and commence at the bottom to add. 4 and 3 are 7 and 2 are 9. Hence the sum is *nine*.

OPERATION.

2
3
4
9

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
2	4	6	1	7	3	1
1	1	0	5	0	4	2
3	3	3	3	2	2	6
6						

(9.)	(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)	(17.)
3	2	4	6	2	3	5	5	3
5	1	2	0	1	4	2	1	2
1	4	3	2	6	2	2	3	4

18. What is the sum of 2, 0, 3, 4?
19. What is the sum of 3, 1, 0, 2, 3?
20. What is the sum of 2, 2, 3, 0, 1?
21. What is the sum of 4, 1, 0, 2, 1?
22. What is the sum of 3, 0, 2, 0, 1, 3?

25. *CLASS II.—Problems of more than one column.*

1. What is the sum of 21, 32, 43?

SOLUTION.—We write the numbers so that units stand under units, and tens under tens, and commence at the right to add. The sum of the units is 3 and 2 are 5 and 1 are 6, which we write in units' place. The sum of the tens is 4 and 3 are 7 and 2 are 9, which we write in tens' place. Hence the sum is 96.

OPERATION.

21
32
43
96

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)
31	20	34	12	15
23	14	20	23	40
24	25	15	54	34
78				

(7.)	(8.)	(9.)	(10.)	(11.)
121	214	610	234	361
213	312	156	432	215
432	153	213	123	123
766				

(12.)	(13.)	(14.)	(15.)	(16.)
612	314	712	416	201
105	212	150	141	305
271	271	137	222	281

(17.)	18.)	(19.)	(20.)	(21.)
2021	5234	6141	7124	6214
3514	1321	1213	1321	322
2361	2141	2032	2042	1211

(22.)	(23.)	(24.)	(25.)	(26.)
34123	41210	50273	1234	23071
14310	13025	17202	4012	12303
20341	21613	21310	3701	20413
11111	12030	10101	1020	21210

Required the sum

27. Of 2031, 1234, 3122, and 1010.
28. Of 1207, 3040, 2430, and 2112.
29. Of 2051, 3027, 1500, and 1320.
30. Of 21021, 2712, 12032, 102, and 21.
31. Of 12201, 23021, 2142, and 12012.

Case II.

26. To add when the sum of any column is more than nine.

27. *CLASS I.—Problems of one column.*

1. What is the sum of 7, 6, and 8?

OPERATION.

7
6
8
21 Ans.

SOLUTION.—We write the numbers one under the other, and commence at the bottom to add. 8 and 6 are 14 and 7 are 21. We place the 1 under the column, and the 2 in tens' place.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
3	7	8	6	8	3
4	2	2	3	2	9
6	5	7	7	6	7

(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
5	7	6	4	7	6
4	3	3	3	4	7
3	8	8	6	7	3
2	5	7	5	5	8

(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
8	3	7	2	4	7
7	7	1	0	5	8
3	2	3	7	6	9
2	5	5	8	7	6
4	5	6	9	8	5

Required the sum

20. Of 6, 7, 5, 3, 2, and 4.
21. Of 3, 2, 7, 4, 6, and 7.
22. Of 3, 4, 5, 6, 7, and 8.
23. Of 4, 5, 6, 2, 3, and 5.
24. Of 2, 7, 3, 1, 4, and 6.
25. Of 3, 5, 4, 3, 2, and 4.
26. Of 3, 6, 7, 2, 1, and 4.
27. Of 1, 3, 2, 7, 8, and 5.
28. Of 8, 2, 4, 6, 5, and 6.
29. Of 3, 6, 5, 3, 7, and 2.
30. Of 4, 3, 2, 5, 6, and 7.
31. Of 6, 2, 7, 4, 5, and 8.

28. *CLASS II.—Problems of more than one column.*

1. What is the sum of 65, 46, and 32?

SOLUTION 1.—We write the numbers units under units and tens under tens, and commence at the right to add. 2 and 6 are 8 and 5 are 13, *units*, which equal 1 *ten* and 3 *units:* we write the 3 units under the column of units, and add the 1 ten to the column of tens. 3 and 1 are 4 and 4 are 8, and 6 are 14, *tens*, which equal 1 *hundred* and 4 *tens;* we write the 4 tens in tens' place, and the 1 hundred in hundreds' place, and we have 143.

OPERATION

65
46
32
143 Ans

SOLUTION 2.—After the pupil is familiar with the above solution he may abbreviate it thus: 2 and 6 are 8 and 5 are 13; we write the 3 and add the 1. One and 3 are 4, and 4 are 8, and 6 are 14 which we write.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
43	27	37	43	58	76
38	56	25	49	36	24
81					

(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
23	28	34	44	82	18
36	51	47	56	17	71
47	35	22	31	45	49

(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
247	462	442	756	361	826
358	379	867	482	484	108

(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
317	424	365	813	678	725
452	536	407	791	123	146
324	817	324	142	414	234

(26.)	(27.)	(28.)	(29.)	(30.)	(31.)
463	282	365	216	417	318
217	187	149	418	282	182
345	208	372	732	479	479

(32.)	(33.)	(34.)	(35.)	(36.)	(37.)
729	321	242	813	183	815
538	467	517	916	517	581
212	213	343	732	648	186
400	457	525	145	422	307

(38.)	(39.)	(40.)	(41.)	(42.)	(43.)
361	217	678	678	489	289
163	721	321	910	201	303
725	548	473	112	232	132
643	918	258	814	425	333
146	172	345	756	267	456

(44.)	(45.)	(46.)	(47.)	(48.)	(49.)
4567	1718	2526	3343	4243	1525
8910	1920	2728	5363	4546	3545
1112	2122	9303	7389	4748	5565
3456	2324	1323	4041	9505	7585

(50.)	(51.)	(52.)	(53.)	(54.)	(55.)
5960	7374	8163	8124	2185	6215
6162	5789	2738	1792	6727	8372
3646	2100	2543	8547	9858	5728
5666	4731	7342	3218	2832	6217
7869	2578	1856	4002	1479	1234

(56.)	(57.)	(58.)
48721	32173	67321
32578	82573	73214
41625	21289	84366
78321	47020	92785
47856	21832	12346

59. Find the sum of 2185+6357+4832+6719+4324.
60. Find the sum of 4344+4647+4849+5051+5253.
61. Find the sum of 6432+7253+2187+6730+5087
62. Find the sum of 2426+3275+8397+2547+8037
63. Find the sum of 234+6721+853+8762+3739.
64. Find the sum of 834+6737+8321+123+9207.
65. Find the sum of 3246+2109+465+3712+2573

66. Find the sum of 8213+123+6785+3282+7654.

67. Find the sum of 123+456+7821+6731+1234.

68. Find the sum of 622+8763+1234+5678+910123

69. Find the sum of 23456+12345+70205+21846+31082.

PRACTICAL PROBLEMS.

1. Mary has 15 apples and John has 23 apples; how many have they both?

SOLUTION.—If Mary has 15 apples and John has 23 apples, they both have the sum of 15 apples and 23 apples, which is 38 apples.

NOTE.—Very young pupils may say, they both have the sum of 15 apples and 23 apples, which is 38 apples.

OPERATION.

15
23
38 Ans.

2. There were 25 robins on one tree and 36 robins on another tree; how many robins were there on both trees?

3. Willie has 36 cents in one pocket and 45 cents in the other; how many has he in both pockets?

4. A little boy had 37 walnuts, and then picked 56 more; how many walnuts did he then have?

5. Emma's doll cost 95 cents, and a little cradle for it cost 225 cents; how much did both cost?

6. There were 48 roses on one bush and 39 roses on another bush; how many roses were there on both bushes?

7. A little girl read 146 words one day and 178 words the next day; how many words did she read both days?

8. Harry had 246 cents in his money-box, and his uncle gave him 175 cents; how many cents had he then?

9. Peter's kite arose 436 feet, and Andrew's kite went 58 feet higher; how high did Andrew's kite arise?

10. Edward took 692 steps in going to school, and

Mary took 742 steps; how many steps did they both take?

11. Mary's garden contains 47 roses, 39 pinks and 52 lilies; how many flowers are in Mary's garden?

12. Sallie spelled 25 words correctly, Jennie 36 words, and Maggie 28 words; how many did they all spell correctly?

13. Charlie wrote 346 words last week and 378 words this week; how many words did he write in the two weeks?

14. Minnie saw 46 swallows in a flock, and Maggie saw 54 swallows in another flock; how many swallows did they both see?

15. Frank says he took 627 steps in going to school, and only 596 steps in coming from school; how many steps did he take in all?

16. My father has 6 horses, 13 cows, and 46 sheep; how many animals has he in all?

17. Emma's new reader contains 46 pictures, and Ella's contains 78 pictures; how many pictures are there in both of these readers?

18. Edward's top cost 25 cents, his whip cost 43 cents, and his ball cost 75 cents; how many cents did they all cost?

19. Albert's father owned 27 little pigs, and Peter's father owned 34 little pigs; how many little pigs had they both?

20. My father gave 215 cents for my cap, 365 cents for my vest, and 625 cents for my coat; how many cents did he give for them all?

21. One old hen had 17 little chickens, another had 15 little chickens, and another 16; how many chickens did the three hens have?

22. Henry learned seventy-five words one week and eighty-four words the next week; how many words did he learn both weeks?

23. Maria has fifty-seven cents in her money-bank, and her aunt put twenty-five cents more in the bank; how many cents did she then have?

24. There were sixteen robins in a tree, twenty-four on the barn, and thirty-nine in the meadow; how many robins were there in all?

25. Julia gave a poor old soldier ninety-six cents, Annie gave him seventy-seven cents, and Carrie gave him one hundred and seventeen cents; how much did the old soldier receive?

PRACTICAL PROBLEMS.

1. A gave 27 dollars for a cow, 45 dollars for an ox, and 150 dollars for a horse; what did they all cost?

2. A has 120 acres of land, B has 310 acres, C has 516 acres, and D has 715 acres; how many acres have they together?

3. There are 31 days in January, 28 in February, 31 in March, and 30 in April; how many days in these four months?

4. A man travelled 215 miles one week, 195 the next week, 273 the next, and 378 the next; how far did he travel?

5. A weighs 127 pounds, B weighs 215 pounds, C 176 pounds, D 184 pounds, and E 234 pounds; what is the sum of their weights?

6. A farmer raised 576 bushels of corn, 918 bushels of oats, 3149 bushels of wheat, and 2785 bushels of rye; how many bushels did he raise in all?

7. A owns 214 acres of land, B owns 719 acres, C owns 2136 acres, and D owns 372 acres; how many acres do they together own?

8. A bought a horse for 168 dollars, and a carriage for 376 dollars, and sold them so as to gain 89 dollars; *what did he* receive?

9. A drover had 327 sheep, 496 cows, 819 pigs, 123 oxen, and 216 horses in his drove; how many animals had he in the drove?

10. There are 39 books and 929 chapters in the Old Testament, and 27 books and 260 chapters in the New Testament; how many are there in both?

11. In an orchard 87 trees bear apples, 26 bear peaches, 38 bear plums, and 17 bear cherries; how many trees are there in the orchard?

12. Mr. Wilson's farm is worth 3720 dollars, his bank stock is worth 1250 dollars, and he has 7257 dollars in money; how much is he worth?

13. A man bought a farm for 7500 dollars, paid 6550 dollars for building a house and barn, and then sold it so as to gain 725 dollars; what did he receive for it?

14. Harvey bought a knife for 37 cents, a hoop for 75 cents, a book for 68 cents, and a top for 87 cents, he sold them at a gain of 23 cents; what did he receive for them?

15. William lends his brother 3275 cents, his sister 4287 cents, his father 3851 cents, and has 4892 cents left; how much money had he?

16. In one book there are 725 pages, in another book there are 327 pages, and in another book there are as many as in both the former; how many pages in all?

17. A merchant bought cloth for 756 dollars, silk for 859 dollars, muslin for 367 dollars, and calico for 255 dollars; how much did they all cost?

18. A paid 325 dollars for a span of horses, and 248 dollars more than this for a carriage; for how much must he sell them both to gain 275 dollars?

19. A gains in one year 465 dollars, B gains 136 dollars more than A, and C gains as much as A and B both; how much did B gain? how much did C gain? how much did they all gain?

SECTION III.

SUBTRACTION.

29. ***Subtraction*** is the process of finding the *difference* between two numbers.

30. The ***Difference,*** or ***Remainder,*** is the number of units more in the greater than in the less.

31. The ***Minuend*** is the number from which we subtract. The ***Subtrahend*** is the number to be subtracted.

32. The ***sign of Subtraction*** is —, and is read *minus.* It denotes that the number after the sign is to be subtracted from the one before it.

CASE I.

33. To subtract when no figure of the subtrahend expresses more units than its figure in the minuend.

34. *CLASS I.—When the subtrahend is one figure.*

1. Subtract 4 from 9.

SOLUTION.—We write the 4 under the 9 and say, 4 units from 9 units leave 5 units, which we write beneath.

OPERATION.

9
4
5 Ans.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
5	7	5	6	8	7
2	3	3	2	5	5
(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
8	7	6	9	8	9
3	4	3	6	2	7

14. Subtract 3 from 9; 6 from 17; 7 from 19; 8 from 19.

15. Subtract 2 from 14; 4 from 9; 8 from 18; 6 from 19.

16. Subtract 7 from 18; 5 from 17; 6 from 18; 5 from 16.

17. Subtract 8 from 19; 5 from 16; 9 from 19; 4 from 17.

35. *CLASS II.—When each term is two or more figures.*

1. Subtract 24 from 67.

SOLUTION.—We write the 24 under the 67, units under units, and tens under tens, and commence at the right to subtract. 4 units from 7 units leave 3 units, 2 tens from 6 tens leave 4 tens; hence the remainder is 4 tens and 3 units, or forty-three.

OPERATION.

67
24
—
43 Ans.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
58	86	72	53	46	76
35	24	41	22	15	24
—	—	—	—	—	—
(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
49	67	85	97	86	99
27	26	52	25	73	25
—	—	—	—	—	—
(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
625	456	763	617	767	896
312	215	512	215	123	432
—	—	—	—	—	—
(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
872	725	857	907	840	876
161	413	654	205	320	345
—	—	—	—	—	—

(26.)	(27.)	(28.)	(29.)	(30.)	(31.)
279	807	796	736	967	875
136	502	452	432	234	345

(32.)	(33.)	(34.)	(35.)	(36.)	(37.)
8763	9076	3769	5076	4872	7659
4321	4054	1546	3075	2342	3237

(38.)	(39.)	(40.)	(41.)	(42.)	(43.)
8769	4876	8275	8799	8591	5857
3257	2142	3251	2542	7230	1234

(44.)	(45.)	(46.)	(47.)	(48.)
82345	57596	72578	27397	67385
22121	21321	41362	22315	24123

(49.)	(50.)	(51.)	(52.)	(53.)
57897	67858	87578	96754	81296
21472	32721	21335	21423	20135

(54.)	(55.)	(56.)	(57.)	(58.)
253786	472589	87695	56728	98785
213123	212423	23542	21306	21342

(59.)	(60.)	(61.)	(62.)	(63.)
373967	873972	72587	95837	89976
212851	132421	51234	51321	32742

Subtract

64. 314 from 678.
65. 425 from 658.
66. 561 from 789.
67. 254 from 576.
68. 437 from 869.

Subtract

69. 1235 from 3768.
70. 3726 from 4969.
71. 2532 from 8748.
72. 4720 from 87856.
73. 12345 from 68799.

Case II.

36. To subtract when a figure in the subtrahend expresses more than the corresponding figure in the minuend.

37. *CLASS I.—When the subtrahend is one figure.*

1. Subtract 8 from 12.

Solution.—We write the 8 under the 12; then 8 from *twelve* is *four*.

OPERATION.

12
8
4 Ans.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)
12	12	13	14	10	11	10	13
9	7	8	6	6	7	8	9
							4

(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)	(17.)
15	15	16	13	16	17	16	14
7	8	9	7	8	9	6	5

(18.)	(19.)	(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
10	17	13	11	10	19	14	16
3	6	5	4	2	9	8	7

38. *CLASS II.—When each term is two or more figures.*

1. Subtract 45 from 82.

Solution 1.—We write the 45 under 82, placing units under units, and tens under tens, and commence at the right to subtract. We cannot subtract 5 units from 2 units; we will therefore take 1 ten from the 8 tens, leaving 7 tens; 1 ten equals 10 units, which added to 2 units equals 12 units; 5 units from 12 units leave 7 units; 4 tens from 7 tens leave 3 tens; hence, the remainder is 37.

OPERATION.

82
45
37 Ans

Solution 2.—We cannot take 5 units from 2 units; we will *therefore add 10 units to the* 2 units, making 12 units; 5 units from 12

units leave 7 units. Now, since we have added 10 units, or 1 ten, to the minuend, our remainder will be 1 ten too large; hence, we must add 1 ten to the subtrahend; 1 ten and 4 tens are 5 tens, 5 tens from 8 tens leave 3 tens.

NOTE.—In practice we solve thus; 5 from 2 we cannot take, but 5 from 12 leaves 7, 4 and 1 are 5, and 5 from 8 leaves 3.

39. From the preceding explanations we have the following general rule.

RULE.—1. *Write the smaller number under the larger, with units under units, tens under tens, etc., and commence at the right to subtract.*

2. *Take the number denoted by each figure of the subtrahend from the number denoted by the corresponding figure of the minuend, and write the result beneath.*

3. *If the number denoted by a figure in the subtrahend is greater than the number denoted by the corresponding figure in the minuend, add 10 to the latter and then subtract, and add 1 to the next left-hand place in the subtrahend.*

40. PROOF.—Add the remainder to the subtrahend; the sum will equal the minuend if the work is correct.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
73	64	32	41	53	62
25	27	14	26	28	28
(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
75	31	57	63	87	95
26	18	29	45	28	59
(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
87	75	63	77	87	94
39	38	25	48	59	49
(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
72	84	70	81	90	97
27	48	17	18	39	79

(26.)	(27.)	(28.)	(29.)	(30.)	(31.)
342	573	692	545	826	357
124	245	457	328	252	183
(32.)	(33.)	(34.)	(35.)	(36.)	(37.)
573	748	835	968	839	538
248	375	573	675	584	394
(38.)	(39.)	(40.)	(41.)	(42.)	(43.)
659	839	547	658	735	848
475	583	284	372	373	539
(44.)	(45.)	(46.)	(47.)	(48.)	(49.)
524	752	845	307	456	450
356	387	579	138	387	382
(50.)	(51.)	(52.)	(53.)	(54.)	(55.)
854	943	607	500	704	403
396	765	309	325	507	285
(56.)	(57.)	(58.)	(59.)	(60.)	(61.)
726	857	735	792	807	650
387	389	558	295	328	357
(62.)	(63.)	(64.)	(65.)	(66.)	(67.)
3876	6385	6735	4076	4070	4135
2379	3527	2547	3128	2137	1216
(68.)	(69.)	(70.)	(71.)	(72.)	(73.)
8672	5283	8175	2534	6735	7219
3728	2426	2836	1235	5376	1972
(74.)	(75.)	(76.)	(77.)	(78.)	(79.)
8522	7135	6347	8135	7345	4372
6243	1872	2563	2453	2876	2583
(80.)	(81.)	(82.)	(83.)	(84.)	(85.)
65672	43763	87253	73875	63527	53413
23828	24235	34365	38376	14238	28401

(86.)	(87.)	(88.)	(89.)	(90.)	(91.)
73285	20307	87004	76500	40500	37201
43836	15231	34523	43654	37254	23534

(92.)	(93.)	(94.)	(95.)	(96.)	(97.)
83030	90304	50310	60204	70000	100000
76543	40372	30311	30205	32463	1

PRACTICAL PROBLEMS.

1. Mary had 25 roses and gave Sarah 12 of them; how many did Mary then have?

SOLUTION.—If Mary had 25 roses and gave Sarah 12 of them, Mary then had the difference between 25 roses and 12 roses, which is 13 roses.

NOTE.—Quite young pupils may say, Mary then had the difference between 25 roses and 12 roses, which is 13 roses.

OPERATION.

25
12
13 ANS.

2. Willie had 34 cents and gave James 18 cents; how many cents did Willie then have?

3. A little girl had 54 pins and gave her cousin 27 of them; how many did she have remaining?

4. Fifty little robins were sitting on a tree, and 23 of them flew away; how many were then left?

5. There were 96 peaches on an old peach-tree, and a gust of wind blew 37 of them off; how many then remained on the tree?

6. Henry's top and ball cost 120 cents; how much did the top cost, if the ball cost 75 cents?

7. Emma's doll and its little cradle cost 320 cents, and the doll cost 95 cents; how much did the cradle cost?

8. There were 87 roses on two rose-bushes; how many roses were there on the second bush, if there were 39 roses on the first bush?

9. A little girl read 324 words in two days; if she *read 146 words* one day, how many did she read the *other day?*

10. Edward and Mary together took 1434 steps in going to school; how many steps did Mary take, if Edward took 692 steps?

11. Minnie had 372 cents in her money-bank, and took out 164 cents to give to a little beggar-girl; how many cents remained?

12. Andrew's kite arose 494 feet, and this was 58 feet higher than Peter's kite went; how high did Peter's kite fly?

13. Charlie wrote 724 words in two weeks; he wrote 346 words the first week; how many words did he write the second week?

14. Mary's new reader contains 76 pictures, and Fanny's contains 92 pictures; how many does Fanny's contain more than Mary's?

15. Two little girls picked 74 quarts of blackberries one summer; if one picked 37 quarts, how many quarts did the other pick?

16. Thomas said he counted 283 crows in his father's cornfield; he threw a stone and scared 126 away; how many then remained?

17. Floy and Eugie together took 3000 steps one day; if Floy took 1786 steps, how many steps did Eugie take?

18. Effie and Eddie counted their chestnuts and found they together had 1232; now, if Eddie had 675, how many had Effie?

19. Mary's mother bought her an arithmetic and slate for 125 cents; if the slate cost 45 cents, what did the arithmetic cost?

20. Herbert's father bought him a cap and coat for 850 cents; he paid 125 cents for the cap; how much did he pay for the coat?

21. Mr. Nelson's horse and carriage cost four hundred dollars; what did the horse cost, if the carriage cost *two hundred and* twenty-five dollars?

22. Two little boys picked eighty-four quarts of black berries one summer; if one picked forty-seven quarts, how many quarts did the other pick?

23. Mr. Barton raised two thousand bushels of wheat and rye; how much rye did he raise, if he raised five hundred and sixty-five bushels of wheat?

PRACTICAL PROBLEMS.

1. A man had 78 cows and sold 24 of them; how many cows remained?

SOLUTION.—If a man had 78 cows and sold 24, there remained the difference between 78 and 24, which we find by subtracting is 54.

NOTE.—Quite young pupils may merely say, there remains the difference between 78 and 24, which is 54.

OPERATION

78
24
54 Ans.

2. A boy had 150 cents and spent 75 cents; how many cents then remained?

3. A man had 325 apples and sold 180 apples; how many had he then?

4. Henry had 1735 dollars, lent his brother 854 dollars; how many dollars remained?

5. A bought 570 horses and sold 295 of them; how many remained unsold?

6. A and B together had 7256 acres of land; how many had B if A had 3627?

7. Two men have 8570 bushels of grain, and the first has 2846 bushels; how many has the second?

8. Washington was born 1732 and died 1799; how old was he at his death?

9. John Adams was born 1735 and died 1826; how old was he at his death?

10. Jefferson was born 1743 and died 1826; how old was he at his death?

11. Madison was born 1758 and died 1836; how old was he at his death?

12. Monroe was born 1758 and died 1831; how old was he at his death?

13. John Quincy Adams was born 1767 and died 1848; how old was he at his death?

14. Jackson was born 1767 and died 1845; how old was he at his death?

15. A has 5480 bushels of oats, which is 975 bushels more than B has; how many bushels has B?

16. In an army of 50000 men 628 were killed and 2596 wounded; how many remained unhurt?

17. A farmer had 234 hens and bought 367, and then sold 489; how many then remained.

18. A merchant sold goods to the amount of 7580 dollars and gained 1396 dollars; what did the goods cost?

19. A and B have each 1840 acres of land; A sells B 895 acres; how many has each then?

20. A farmer has 1346 sheep and 849 lambs; how many more sheep has he than lambs?

21. Mary and Eliza have each 789 cents; if Eliza gives Mary 247 cents, how many will each then have?

22. Subtract six hundred and seventy-eight from nine hundred and four.

23. Add seven hundred and fifteen to five hundred and seventy-three, and subtract the sum from two thousand.

24. Find the sum of one thousand and ninety-six and five hundred and forty-five, and subtract it from three thousand.

25. Frank solved four hundred and sixteen problems, and Fanny solved five hundred and three problems; how many did Fanny solve more than Frank?

26. A farmer had 2346 bushels of wheat; he sold one man 687 bushels and another man 1560 bushels; how many bushels did he sell? how many remained?

PRACTICAL PROBLEMS in Addition and Subtraction.

1. If I have 75 cents in my money-bank, and my uncle puts in 26 cents, how much will be in it then?

2. If Willie reads 125 words this week and 187 words next week, how many words will he read in all?

3. My father had 236 little chickens, and a mink killed 48 of them; how many remained?

4. If I have 438 dollars and give my sister 246 dollars, how much will I have remaining?

5. Mary's father had 360 acres of land and sold 125 acres; how many acres did he then have?

6. Peter had 467 dollars and lent his brother 185 dollars; how much did he then have?

7. I have 365 cents in my money-bank; how many must I put in that there may be 400 cents in it?

8. Sallie had 72 cents and her brother gave her enough to make her money 134 cents; how much did her brother give her?

9. Carrie's brother teased her because she couldn't tell how many she must add to 245 to make 400; can you tell?

10. Matilda had 120 cents, her mother gave her 236 cents, and then she lent her brother 248 cents; how many cents did she then have?

11. Fannie picked 236 chestnuts, her little brother gave her 78 chestnuts, and she gave 95 to her schoolmates; how many chestnuts remained?

12. Mary cried because she couldn't tell her teacher how many she must add to 367 to make 500; tell me how many it is.

13. One morning in going to school I took 726 steps; how many more would I have taken if I had taken 1000 in all?

14. My kite was up in the air 436 feet, it then fell *185 feet, and then* arose 260 feet; how high was it then?

BUSINESS PROBLEMS.

1. I went to a store and bought a book for 87 cents and a slate for 35 cents; what did I pay for both of them?

2. My mother took me to a store and bought me a top for 15 cents, a cap for 75 cents, and a knife for 45 cents; what did they all cost?

3. William's slate cost 26 cents, his arithmetic 55 cents, his reading-book 48 cents, and his spelling-book 37 cents; what did they all cost?

4. I went to a store and bought a knife for 56 cents and gave the storekeeper a dollar bill (100 cents) to pay for it; how much change did he give me back?

5. Mary bought a flower-vase for 375 cents, and handed the storekeeper a five-dollar bill (500 cents) to pay for it; how much change should she have received?

6. Mr. Barnes paid 75 dollars for his watch, and sold it so that he gained 12 dollars; what did he receive for it?

7. Martha's new shawl cost 875 cents; if she should sell it so as to gain 125 cents, what would she receive for it?

8. Mr. Taylor's new house cost him 3675 dollars, and he sold it for 565 dollars more than it cost him; what did he receive for it?

9. My father bought a cow for 38 dollars and sold her for 52 dollars; how much did he gain on the cow?

10. Robert Stewart had a coat which cost him 45 dollars; he sold it to Edward Taylor for 37 dollars; how much did he lose?

11. Harry Hartman sold his watch for 67 dollars and lost by the sale 15 dollars; what did the watch cost him?

12. Mary's papa gave her a 5 dollar bill to go a shopping; she bought a fan for 75 cents, some silk for 165 cents, and a pair of gloves for 125 cents; how much change did she bring home?

PRACTICAL PROBLEMS
in Addition and Subtraction.

1. What is the value of 675 + 432 + 285 + 672?
2. What is the value of 362 + 486 + 721 — 367?
3. What is the value of 473 + 325 + 604 — 1206?
4. What is the value of 3072 + 4861 + 2075 — 6785?
5. Subtract 1678 from the sum of 985 and 863.
6. Subtract the sum of 265 and 381 from the sum of 281 and 678.
7. Subtract 218 + 318 + 418 from 379 + 279 + 479.
8. A having 475 dollars earned 220 dollars and then spent 567; how much remained?
9. Newspapers were first published in 1630; how long have they been published?
10. Quills were first used for writing about the year 636; how long is it since?
11. Cotton was first planted in the United States about the year 1769; how many years since?
12. Glass windows, it is said, were first used in England in 1180; how long is it since then?
13. A sold his farm for 12450 dollars, which was 1680 dollars more than it cost; how much did it cost?
14. A gave 6500 dollars for his farm and 2560 dollars for his house, and sold them for 12000; what was the gain?
15. A farmer had 5600 bushels of corn, and sold 1850 bushels to A and 2810 to B; how much remained?
16. The area of Maine is 30000 square miles, and of New York 46000; how much larger is New York than Maine?
17. The area of Massachusetts is 7800 square miles, and of Pennsylvania 46000 square miles; how much larger is Pennsylvania than Massachusetts?
18. How much larger are Pennsylvania and Maine *together than* New York and Massachusetts together?

MULTIPLICATION.

41. ***Multiplication*** is the process of finding the result of taking one number as many times as there are units in another.

42. The ***Multiplicand*** is the number to be multiplied.

43. The ***Multiplier*** is the number by which we multiply.

44. The ***Product*** is the result obtained.

44½. ***The sign of Multiplication*** is $\times$, and is read *multiplied by:* thus, $4 \times 3 = 12$ means 4 *multiplied by* 3 *equals* 12. The 4 is the multiplicand, 3 is the multiplier, and 12 is the product.

NOTE TO TEACHERS.—If the pupils are not familiar with the Multiplication Table, let them now turn to page 20 and learn it.

CASE I.

45. When the multiplier is one figure.

CLASS I.—When no product exceeds nine.

1. Multiply 34 by 2.

SOLUTION.—We write the multiplier under the multiplicand, and begin at the right to multiply. 2 times 4 units are 8 units; we write the 8 units in units' place. 2 times 3 tens are 6 tens; we write the 6 tens in tens' place.

OPERATION.

34
2
—
68 Ans.

(2.)	(3.)	(4.)	(5.)
32	24	14	41
2	2	2	2
—	—	—	—
(6.)	(7.)	(8.)	(9.)
21	12	23	32
3	3	3	3
—	—	—	—

46. *CLASS II.—When some of the products exceed nine.*

1. Multiply 56 by 4.

SOLUTION 1.—We write the multiplier under the multiplicand and begin at the right to multiply. 4 times 6 units are 24 *units*, which equal 4 units and 2 tens. We write the 4 units in units place, and reserve the 2 tens to add to the next product. 4 times 5 tens are 20 tens, plus the 2 tens equal 22 tens, which equal 2 tens and 2 hundreds, which we write in their proper places. Hence the product is 224.

OPERATION.

56
4
224 Ans.

SOLUTION 2.—4 times 6 are 24; we write the 4 and add the 2 to the next product. 4 times 5 are 20, and 2 added equal 22; hence the product is 224. From this we have the following

RULE.— *Write the multiplier under the multiplicand, draw a line beneath, begin at units, and multiply the number denoted by each figure of the multiplicand by the multiplier, carrying as in addition.*

(2.)	(3.)	(4.)	(5.)	(6.)
25	36	47	73	28
3	2	3	2	4
(7.)	(8.)	(9.)	(10.)	(11.)
63	75	36	27	43
5	4	5	6	5
(12.)	(13.)	(14.)	(15.)	(16.)
75	86	92	76	84
2	3	4	5	6
(17.)	(18.)	(19.)	(20.)	(21.)
73	47	76	85	73
5	6	7	8	8
(22.)	(23.)	(24.)	(25.)	(26.)
234	425	673	723	351
3	4	5	6	7
(27.)	(28.)	(29.)	(30.)	(31.)
425	314	421	636	854
6	7	8	7	8

(32.)	(33.)	(34.)	(35.)	(36.)
256	375	873	358	725
4	6	7	8	7

(37.)	(38.)	(39.)	(40.)	(41.)
581	809	394	908	765
7	8	6	9	8

Multiply	Multiply
42. 3124 by 4.	50. 13257 by 2.
43. 2856 by 5.	51. 36072 by 3.
44. 7863 by 6.	52. 85761 by 4.
45. 2185 by 7.	53. 35167 by 5.
46. 4182 by 8.	54. 84307 by 6.
47. 3075 by 8.	55. 30754 by 7.
48. 4107 by 9.	56. 21836 by 8.
49. 7685 by 6.	57. 35168 by 9.

Case II.

47. When the multiplier consists of two or more figures.

48. *CLASS I.—When the multiplier consists of two figures.*

1. Multiply 64 by 23.

OPERATION.

64
23
192
128
1472 Ans.

Solution 1.—We write the multiplier under the multiplicand, placing units under units, and tens under tens, and begin at the right to multiply. 3 times 4 units are 12 units, which equals 1 ten and 2 units; we write the units under the 3, and reserve the 1 ten to add to the next product. 3 times 6 tens are 18 tens, and 1 ten added equals 19 tens, or 1 hundred and 9 tens, which we write in their proper places. Multiplying 64 by 2 in the same manner, we have 128, and since the 2 is 2 *tens* we have 128 tens, which we write in its proper place; then, adding the two products, we have 1472.

Solution 2.—Three times 4 are 12; we write the 2 and carry the

1: three times 6 are 18, plus the 1 equals 19; which we write Then, 2 times 4 are 8, which we write under the 2, and 2 times 6 are 12, which we write beside the 8.

RULE.—1. *Write the multiplier under the multiplicand, placing units under units, tens under tens etc., and begin at the right to multiply.*

2. *Multiply the multiplicand by the number denoted by each figure of the multiplier, writing the first figure of each product under the figure of the multiplier used.*

3. *Add together the partial products, and their sum will be the entire product.*

PROOF.—Multiply the multiplier by the multiplicand; if the two results agree, the work is probably correct

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
38	43	73	81	29	57
23	24	35	67	82	75

(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
87	39	87	29	123	245
28	43	52	92	37	32

(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
436	534	427	426	534	672
43	43	35	43	45	46

(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
725	634	807	475	709	493
42	47	37	54	38	82

(26.)	(27.)	(28.)	(29.)	(30.)	(31.)
756	762	675	467	762	812
93	48	39	37	62	45

(32.)	(33.)	(34.)	(35.)	(36.)	(37.)
1234	2341	6724	6357	7138	2536
28	35	42	35	52	25

(38.)	(39.)	(40.)	(41.)	(42.)	(43.)
6347	8192	4736	4825	3121	4073
46	73	63	72	37	46

Multiply	Multiply
44. 6538 by 83.	49. 4175 by 28.
45. 7384 by 45.	50. 7186 by 85.
46. 2185 by 67.	51. 8391 by 94.
47. 3407 by 82.	52. 2187 by 89.
48. 3584 by 46.	53. 6543 by 98.

49. *CLASS II.—When the multiplier consists of three figures.*

(1.)	(2.)	(3.)	(4.)	(5.)
4126	5731	7351	1375	5379
234	243	432	342	423

(6.)	(7.)	(8.)	(9.)	(10.)
6725	2183	7321	8193	2147
345	544	265	475	813

(11.)	(12.)	(13.)	(14.)	(15.)
2143	8192	2435	4167	8246
227	426	146	245	642

(16.)	(17.)	(18.)	(19.)	(20.)
7346	7516	8927	4928	2076
643	571	352	816	437

(21.)	(22.)	(23.)	(24.)	(25.)
4752	7385	8492	2937	6473
185	218	537	439	567

Multiply	Multiply
26. 14651 by 283.	35. 28352 by 345.
27. 31251 by 625.	36. 41678 by 287.
28. 36782 by 234.	37. 34073 by 435.
29. 43678 by 452.	38. 40735 by 628.
30. 36507 by 634.	39. 29304 by 789.
31. 40725 by 365.	40. 90705 by 897.
32. 32107 by 681.	41. 43445 by 678.
33. 25697 by 329.	42. 37436 by 835.
34. 42046 by 456.	43. 88888 by 789.

50. *CLASS III.—When the multiplier consists of more than three figures.*

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
4137	3642	6724	4183	3645	4526
2185	2531	3625	2426	2841	2182

(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
3482	2846	3707	4172	2882	8567
2534	2528	2851	2174	2773	3178

(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
5185	9187	4785	8197	4376	8765
8763	2567	7372	1846	5273	5678

Multiply	Multiply
19. 28751 by 3146.	23. 72509 by 3167.
20. 17346 by 2435.	24. 85216 by 2431.
21. 21307 by 3147.	25. 73519 by 4735.
22. 85276 by 3452.	26. 81897 by 3456.

Multiply	Multiply
27. 21346 by 31452.	31. 10786 by 31672.
28. 47309 by 45233.	32. 47396 by 73462.
29. 25737 by 63252.	33. 76448 by 54173.
30. 43629 by 28516.	34. 28354 by 31867.

51. *CLASS IV.—When one or both terms contain ciphers.*

1. Multiply 5721 by 3006; also, 37000 by 2400.

OPERATION.	OPERATION.
5721	37000
3006	2400
34326	148
17163	74
17197326	88800000

NOTE.—In the first example, pass over the naughts, placing the right-hand figure of the product by 3 directly under the 3. In the second problem, we multiply by the significant figures, and then annex the naughts to the product.

Multiply	Multiply
2. 3678 by 204.	11. 4500 by 2800.
3. 4107 by 307.	12. 67000 by 450.
4. 4178 by 1005.	13. 96000 by 2800.
5. 8675 by 3007.	14. 87000 by 4800.
6. 7276 by 6008.	15. 73500 by 32000.
7. 4136 by 2305.	16. 86700 by 47200.
8. 8449 by 3046.	17. 32800 by 346000.
9. 4592 by 5607.	18. 70900 by 407100.
10 8124 by 4801.	19. 85900 by 1030600.

20. *Multiply* four thousand six hundred and ten by *seven thousand* and forty.

EXAMPLES IN MULTIPLICATION.

1. If one orange cost 8 cents, what will 7 oranges cost at the same rate?

SOLUTION.—If one orange cost 8 cents, 7 oranges will cost 7 times 8 cents, which are 56 cents.

OPERATION.

8
7
56 Ans.

2. If one pig cost 7 dollars, what will 6 pigs cost at the same rate?

3. If a yard of muslin cost 37 cents, what will 8 yards cost at the same rate?

4. If a boy writes 36 words in a day, how many will he write in 13 days?

5. What must I pay for 15 cows, if I pay 28 dollars for each cow?

6. If Henry takes 42 steps in a minute, how many steps will he take in 15 minutes?

7. If a car runs 25 miles in an hour, how far will it run in 12 hours?

8. If a boy learns 14 new words each day, how many will he learn in 11 days?

9. Mary has 14 rose-bushes in her garden, and on each bush there are 26 roses; how many roses on all?

10. How much must I pay for 16 pounds of tea, at the rate of 78 cents a pound?

11. What are nine loads of hay worth, at the rate of 23 dollars a load?

12. If one cord of wood is worth six dollars, how much are 18 cords of wood worth?

13. How many marbles will 7 boys have, if each boy has 12 marbles?

14. At the rate of 45 miles a day, how far will a person travel in 23 days?

15 If Henry can count 65 in a minute, how many can he count in 26 minutes?

PRACTICAL EXAMPLES
in Multiplication.

1. What cost 24 horses at 245 dollars each?

SOLUTION.—If one horse cost 245 dollars, 24 horses cost 24 times 245 dollars, which by multiplying we find to be 5880 dollars.

2. If a boat sails 246 miles in one day, how far will it sail in 26 days?

3. If in one book there are 364 pages, how many pages in 18 such books?

4. In one barrel of flour there are 196 pounds; how many pounds in 25 barrels of flour?

5. How much will 42 horses cost, at the rate of 150 dollars apiece?

6. If an acre of land is worth 218 dollars, how much will 76 acres cost?

7. If in an orchard there are 32 rows of trees with 46 trees in a row, how many trees in all?

8. A man bought 326 horses and 36 times as many sheep; how many sheep did he buy?

9. What cost 125 yards of cloth at the rate of 325 cents a yard?

10. How much will 236 bushels of wheat cost, at 175 cents a bushel?

11. There are 1760 yards in one mile; how many yards in 12 miles?

12. There are 5280 feet in a mile; how many feet in 18 miles?

13. There are 660 feet in one furlong; how many feet in 26 furlongs?

14. There are 5760 grains in one pound Troy; how many grains in 137 pounds?

15. There are 256 drams in an ounce; how many drams in 420 ounces?

16. There are 198 inches in one rod; how many inches in 76 rods?

17. There are 1728 pins in a great gross; how many pins in 256 great gross?

18. There are 231 cubic inches in a wine gallon; how many inches in 48 wine gallons?

19. There are 282 cubic inches in a beer gallon; how many cubic inches in 345 beer gallons?

20. There are 4840 square yards in one acre; how many square yards in 365 acres?

21. There are 5280 feet in a mile; how many feet in 156 miles?

22. There are 63360 inches in a mile; how many inches in 640 miles?

23. There are 5280 feet in a mile; how many feet in the diameter of the earth, if it is 7912 miles?

PRACTICAL PROBLEMS
in Multiplication.

1. How much cost 75 barrels of flour, at 7 dollars a barrel?

SOLUTION.—If 1 barrel cost 7 dollars, 75 barrels will cost 75 times 7 dollars, which are 525 dollars.

OPERATION.

75
7
525 Ans.

NOTE.—In practice, we multiply the 75 by 7, since it is more convenient to use the smaller number as the multiplier.

2. How much will 436 bushels of potatoes cost, at 48 cents a bushel?

3. How much will 847 bushels of corn cost, at 56 cents a bushel?

4. How much will 936 yards of muslin cost, at 37 cents a yard?

5. A drover bought 4896 pigs, at 9 dollars each; what did they cost?

6. How much will 3686 grammars cost, at 54 cents apiece?

7. *At 6* cents a quart, what will 3678 quarts of milk cost? *9876* quarts?

8. There are 60 seconds in one minute; how many seconds in 6725 minutes? In 9860 minutes?

9. If 27 men do a piece of work in 48 days, how long will it take one man to do it?

10. If 145 men do a piece of work in 246 days, how long at this rate would it take one man?

11. If 29 men build a fence in 276 days, how long would it take one man to do it?

12. If 200 acres of corn can be hoed by 157 boys in 19 days, how long would it take one boy?

13. If 358 men cut 700 cords of wood in 179 days, how long would it take one man to do it?

14. There are 16536 letters in a book; how many letters in 496 of the same books?

15. If sound moves 1120 feet in one second, how far will it move in 9872 seconds?

16. If the earth moves in its orbit 1640000 miles in a day, how many miles does it move in 365 days?

17. The moon is 240000 miles from the earth, and the sun about 396 times as far; how far is the sun from the earth?

PRACTICAL PROBLEMS
in Addition, Subtraction, and Multiplication.

(Quite young pupils may omit these until review.)

1. A has 245 acres of land, and B has 3 times as much; how many acres has B? how many acres have both?

2. One farmer has 476 hens, and another farmer has 5 times as many, minus 392 hens; how many has the second farmer?

3. One ship sailed 1248 miles, and another sailed 8 times as far, lacking 697 miles; how many miles did the second ship sail?

4. *A has* 485 dollars, and B has 692 dollars; how

nuch money have they both? how much has C, if he ıas 7 times as much as both?

5. Mr. Shank has 6450 bushels of corn, and Mr. Frantz ıas 16 times as much, minus 24986 bushels; how many)ushels has Mr. Frantz?

6. A has 456 dollars, B has 759 dollars, and C has 25 ;imes as much as both, minus 8965 dollars; how many lollars have A and B? how many has C?

7. A man sold 24 cows at 35 dollars each, and 17 ıorses at 275 dollars each; what did he receive for his ;ows? for his horses? for all?

8. A man sold his house for 4560 dollars, and 148 ıcres of land at 245 dollars an acre; how much did he eceive for his house and land?

9. A bought 126 pigs at 8 dollars each, and B bought 7 sheep at 12 dollars each; which cost the most, and ıow much?

10. B bought a house for 2960 dollars, and gave for it 8 cows at 24 dollars each, and the rest in money; how nuch money did he pay?

11. One army contains 4575 men, and another 36 imes as many, lacking 1936 men; how many men in he second army?

12. Mr. Peters has 2461 gallons of coal oil, Mr. Martin ıas 1146 gallons, and Mr. Benson has 147 times as much s both; how much has Mr. Benson?

13. A farmer sold 129 cows at 37 dollars each, and received in payment 2000 dollars; how much yet remains ue?

14. B sold 76 hens at 73 cents each, 96 turkeys at 324 ents each, and received in payment 24000 cents; how ıuch remains due?

15. A's barn cost 2485 dollars, his house cost 3 times s much, and his farm cost as much as both; what was he cost of the house? what was the cost of the farm?

16. A drover bought 36 horses at 145 dollars a head,

and 96 cows at 28 dollars a head; which cost the most, and how much?

17. A's book contains 248 pages, with 2850 letters on a page, and B's contains 325 pages, with 3465 letters on a page; how many letters in A's book? how many in B's?

18. A man has 75 bags of apples, each bag containing 2 bushels; how much will he receive for them, at 125 cents a bushel?

19. A farmer sold 25 firkins of butter, each firkin containing 126 pounds, and received for each pound 37 cents; how much did he receive for it all?

DIVISION.

52 ***Division*** is the process of finding how many times one number is contained in another.

53. The ***Dividend*** is the number which contains the other.

54. The ***Divisor*** is the number contained in the dividend.

55. The ***Quotient*** is the number which shows how many times the dividend contains the divisor.

56. ***The sign of Division*** is ÷, and is read *divided by*. It shows that the number on the left is to be divided by the one on the right.

57. There are two methods of performing division called ***Short Division*** and ***Long Division.***

NOTE TO TEACHERS.—If the pupils are not familiar with the elementary quotients, let them turn to page 21 and learn them.

SHORT DIVISION.

58. ***Short Division*** is the method of dividing when *the partial dividends* are not written.

Case I.

59. When the divisor is one figure.

1. How many times is 2 contained in 6?

Solution 1.—We write the 6, draw a line beneath and a curve to the left, and place the 2 to the left of the curve. *Two* is contained in 6, *three* times, since 3 times 2 are 6. We write the quotient 3 beneath the dividend.

Operation.

$$2)\overline{6}$$
$$\overline{3}$$

Solution 2.—Two is contained in 6 three times, with no remainder.

(2.) 2)8	(3.) 3)6	(4.) 2)4	(5.) 3)9	(6.) 4)8	(7.) 2)10
(8.) 2)12	(9.) 3)12	(10.) 2)14	(11.) 3)18	(12.) 2)20	(13.) 2)22
(14.) 2)24	(15.) 3)21	(16.) 3)27	(17.) 3)30	(18.) 3)36	(19.) 3)33
(20.) 4)16	(21.) 4)24	(22.) 4)28	(23.) 4)20	(24.) 4)36	(25.) 4)48
(26.) 5)15	(27.) 5)25	(28.) 5)35	(29.) 5)20	(30.) 5)40	(31.) 5)55
(32.) 5)60	(33.) 6)12	(34.) 6)24	(35.) 6)36	(36.) 6)48	(37.) 6)60
(38.) 7)21	(39.) 7)35	(40.) 7)49	(41.) 7)63	(42.) 7)77	(43.) 7)84
(44.) 8)16	(45.) 8)64	(46.) 8)56	(47.) 8)40	(48.) 8)72	(49.) 8)96
(50.) 9)27	(51.) 9)45	(52.) 9)63	(53.) 9)81	(54.) 9)108	(55.) 9)99

Case I.

60. When the divisor is one figure and there are no remainders.

1. Divide 46 by 2.

Solution 1.—2 is contained in 4 tens 2 tens times. 2 is contained in 6 units 3 units times; hence the quotient is 23.

Operation.

$$2\overline{)46}$$
$$23$$

Solution 2.—2 is contained in 4, 2 times; 2 is contained in 6, 3 times.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
2)42	2)48	2)26	2)64	2)84	2)86
(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
2)82	3)36	3)69	3)96	3)90	3)39
(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
4)48	4)44	4)88	4)40	4)80	5)50

(20.)	(21.)	(22.)	(23.)	(24.)
2)428	2)228	2)848	2)408	3)369
(25.)	(26.)	(27.)	(28.)	(29.)
3)693	3)906	3)609	3)930	4)480
(30.)	(31.)	(32.)	(33.)	(34.)
4)804	4)408	3)669	4)488	2)880
(35.)	(36.)	(37.)	(38.)	(39.)
2)804	3)936	2)886	2)468	3)603

Case III.

61. When the divisor is one figure and there are remainders.

1. Divide 7 by 3.

SOLUTION 1.—Three is contained in 7, 2 times, which we write under the 7; and since 2 times 3 are 6, and 7 is 1 more than 6, hence 3 is contained in 7, 2 times, and 1 remaining, which we write after the 2 with the sign + before it.

OPERATION.

3(7

2 + 1

SOLUTION 2.—3 is contained in 7, 2 times. 2 times 3 are 6, 6 from 7 leaves 1; hence the quotient is 2, with a remainder of 1.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
2)9	2)11	2)19	2)21	2)13	2)25
(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
3)5	3)11	3)8	3)14	3)17	3)23
(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
4)11	4)17	4)22	4)37	4)43	4)27
2					
(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
5)12	5)19	5)28	5)38	5)47	5)58
(26.)	(27.)	(28.)	(29.)	(30.)	(31.)
6)15	6)21	6)35	6)51	6)65	6)71
(32.)	(33.)	(34.)	(35.)	(36.)	(37.)
7)23	7)29	7)38	7)46	7)58	7)80
(38.)	(39.)	(40.)	(41.)	(42.)	(43.)
8)23	8)19	8)28	8)36	8)47	8)93
(44.)	(45.)	(46.)	(47.)	(48.)	(49.)
9)31	9)26	9)52	9)61	9)70	9)83

CASE IV.

62. When the quotient contains several figures and there are successive remainders.

1. Divide 536 by 2.

SOLUTION 1.—2 is contained in 5 hundreds 2 hundreds times, with 1 hundred remaining; 1 hundred equals 10 tens, which, with 3 tens, equal 13 tens; 2 is contained in 13 tens 6 tens times, and 1 ten remaining; 1 ten equals 10 units, which, with 6 units, equal 16 units; 2 is contained in 16 units 8 units times. Hence the quotient is 268.

OPERATION
2)536
268

SOLUTION 2.—2 is contained in 5, 2 times, and 1 remaining; 2 is contained in 13, 6 times, and 1 remaining; etc.

RULE.—1. *Write the divisor at the left of the dividend; begin at the left hand, and divide the number denoted by each figure of the dividend by the divisor, and write the quotient beneath.*

2. *If there is a remainder after any division, regard it as prefixed to the next figure, and divide as before. If any partial dividend is less than the divisor, prefix it to the next figure, and write a cipher in the quotient.*

63. PROOF.—Multiply the quotient by the divisor, and add the remainder, if any, to the product.

(2.)	(3.)	(4.)	(5.)	(6.)
2)456	2)736	2)548	2)374	2)538
(7.)	(8.)	(9.)	(10.)	(11.)
3)735	3)816	3)522	3)414	3)738
(12.)	(13.)	(14.)	(15.)	(16.)
3)567	3)513	3)645	3)765	3)825
(17.)	(18.)	(19.)	(20.)	(21.)
4)512	4)624	4)732	4)576	4)824
(22.)	(23.)	(24.)	(25.)	(26.)
4)736	4)816	4)972	4)608	4)436
(27.)	(28.)	(29.)	(30.)	(31.)
5)615	5)735	5)645	5)785	5)840
(32.)	(33.)	(34.)	(35.)	(36.)
5)815	5)935	5)780	5)765	5)980

(37.)	(38.)	(39.)	(40.)	(41.)
6)834	6)738	6)654	6)774	6)864
(42.)	(43.)	(44.)	(45.)	(46.)
6)1476	6)3336	6)2514	6)3654	6)7236
(47.)	(48.)	(49.)	(50.)	(51.)
7)2569	7)4732	7)8456	7)9359	7)9870

Divide	Divide
52. 8256 by 8.	59. 72352 by 8.
53. 7656 by 8.	60. 23769 by 9.
54. 9576 by 8.	61. 73145 by 5.
55. 9874 by 9.	62. 5882597 by 7.
56. 9756 by 9.	63. 1101032 by 8.
57. 9387 by 9.	64. 21820708 by 4
58. 92565 by 9.	65. 6328476 by 9.

LONG DIVISION.

64. ***Long Division*** is the method of dividing when the partial dividends are written.

Case I.

65. When the divisor and quotient are each one figure.

1. Divide 7 by 2.

Solution 1.—2 is contained in 7 *three* times. We place the 3 at the right in the quotient, and multiply the divisor by it. 3 times 2 are 6, which we write under the 7. We then draw a line beneath, and subtract, and have 1 remaining.

Operation.

2)7(3
6
1

Solution 2.—2 is contained in 7, 3 times; 3 times 2 are 6, 6 from 7 leaves 1; hence the quotient is 3, and 1 remaining.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
2)5(	2)9(	2)10(	2)12(	2)15(	2)18(

(8.) 2)13(	(9.) 2)11(	(10.) 3)6(	(11.) 3)9(	(12.) 3)8(	(13.) 3)12(
(14.) 3)18(	(15.) 3)21(	(16.) 3)17(	(17.) 3)19(	(18.) 3)23(	(19.) 3)27(
(20.) 4)8(	(21.) 4)12(	(22.) 4)20(	(23.) 4)28(	(24.) 4)30(	(25.) 4)10(
(26.) 4)13(	(27.) 4)23(	(28.) 4)27(	(29.) 5)10(	(30.) 4)20(	(31.) 5)25(
(32.) 5)45(	(33.) 5)35(	(34.) 5)27(	(35.) 5)38(	(36.) 5)43(	(37.) 5)47(
(38.) 6)12(	(39.) 6)24(	(40.) 6)34(	(41.) 6)50(	(42.) 6)59(	(43.) 6)49(
(44.) 7)28(	(45.) 7)49(	(46.) 7)50(	(47.) 7)60(	(48.) 7)48(	(49.) 7)57(
(50.) 8)24(	(51.) 8)37(	(52.) 8)70(	(53.) 8)69(	(54.) 8)59(	(55.) 8)76(
(56.) 9)27(	(57.) 9)63(	(58.) 9)57(	(59.) 9)70(	(60.) 9)76(	(61.) 9)89(

Case II.

66. When the divisor is one figure and the quotient is several figures.

1. Divide 867 by 3.

Solution 1.—3 is contained in 8 hundreds 2 hundreds times. 2 hundreds times 3 equal 6 hundreds. 6 hundreds from 8 hundreds leave 2 hundreds. 2 hundreds and 6 tens are 26 tens. 3 is contained in 26 tens 8 tens times. 8 tens times 3 are 24 tens. 24 tens from 26 tens leave 2 tens. 2 tens and 7 units are 27 units. 3 is contained in 27 units 9 times. 9 times 3 are 27. *Subtracting, nothing* remains. Hence, the quotient is 289.

OPERATION.

```
3)867(289
  6
  --
  26
  24
  --
   27
   27
   --
```

SOLUTION 2.—3 is contained in 8, 2 times; 2 times 3 are 6; 6 from 8 leaves 2. Bring down the 6, and we have 26. 3 is contained in 26, 8 times; 8 times 3 are 24; 24 from 26 leaves 2. Bring down the 7, and we have 27. 3 is contained in 27, 9 times; 9 times 3 are 27, etc.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)
2)36(	2)58(	2)54(	2)92(	2)97(
(7.)	(8.)	(9.)	(10.)	(11.)
3)576(	3)465(	3)723(	3)873(	3)675(
(12.)	(13.)	(14.)	(15.)	(16.)
4)852(	4)764(	4)932(	4)576(	4)748(
(17.)	(18.)	(19.)	(20.)	(21.)
5)735(	5)850(	5)975(	5)745(	5)835(
(22.)	(23.)	(24.)	(25.)	(26.)
6)732(	6)846(	6)924(	6)972(	6)834(
(27.)	(28.)	(29.)	(30.)	(31.)
7)784(	7)798(	7)833(	7)966(	7)959(
(32.)	(33.)	(34.)	(35.)	(36.)
8)896(	8)936(	8)944(	8)976(	8)992(

Divide	Divide
37. 37596 by 2.	46. 46542 by 3.
38. 57672 by 3.	47. 785641 by 6.
39. 78908 by 4.	48. 218030 by 8.
40. 93546 by 6.	49. 51600 by 4.
41. 73455 by 5.	50. 84507 by 7.
42. 75448 by 8.	51. 61243 by 2.
43. 45794 by 7.	52. 47065 by 5.
44. 36783 by 9.	53. 31696 by 6.
45. 487652 by 7.	54. 20040 by 9

Case III.

67. When the divisor is two or more figures.

1. Divide 442 by 13.

Solution.—13 is contained in 44 tens 3 tens times; 3 tens times 13 equal 39 tens; 39 tens from 44 tens leave 5 tens, and bringing down the 2 units we have 52 units. 13 is contained in 52 units 4 times. 4 times 13 are 52; subtracting, nothing remains. Hence the quotient is 34.

Operation.

13)442(34
39
52
52

Note.—With young pupils, abbreviate the explanation, as in the previous solutions.

Rule.—1. *Divide the number expressed by the least number of figures on the left that will contain the divisor, and place the quotient on the right.*

2. *Multiply the divisor by this quotient; write the product under the partial dividend, and subtract, and to the remainder annex the next figure of the dividend.*

3. *Divide as before until all the figures of the dividend have been brought down and used.*

4. *If any partial dividend will not contain the divisor, place a cipher in the quotient, annex the next figure of the dividend, and proceed as before.*

68. Proof.—Multiply the quotient by the divisor, and add the remainder, if any, to the product.

Notes.—1. The pupils will notice that there are four operations: 1st, *Divide*, 2d, *Multiply*, 3d, *Subtract*, 4th, *Bring down*.

2. If when we multiply the product is greater than the partial dividends, the quotient figure is too large, and must be diminished.

3. When a remainder is equal to or greater than the divisor, the quotient figure is too small, and must be increased.

4. A final remainder may be set off by itself, or it may be written over the divisor and annexed to the quotient.

EXAMPLES FOR PRACTICE.

Divide

2. 364 by 11.
3. 780 by 12.
4. 312 by 13.
5. 322 by 14.
6. 570 by 15.
7. 752 by 16.
8. 425 by 17.
9. 594 by 18.
10. 608 by 19.
11. 945 by 21.
12. 2760 by 22.
13. 2852 by 23.
14. 3168 by 24.
15. 5575 by 25.
16. 6396 by 26.
17. 6777 by 27.
18. 10136 by 28.
19. 11948 by 29.
20. 19778 by 31.
21. 16864 by 32.
22. 10725 by 33.
23. 20808 by 34.
24. 7875 by 35.
25. 20616 by 36.
26. 41602 by 37.
27. 39790 by 38.
28. 48725 by 39.
29. 67314 by 41.
30. 82307 by 42.
31. 57256 by 43.
32. 49378 by 44.
33. 98716 by 45.
34. 60904 by 46.
35. 76704 by 47.

Divide

36. 62377 by 49.
37. 84309 by 57.
38. 92736 by 83.
39. 41875 by 123.
40. 1067500 by 500.
41. 1320594 by 561.
42. 1048788 by 468.
43. 932694 by 371.
44. 1011312 by 432.
45. 1459865 by 605.
46. 1612072 by 616.
47. 1704186 by 627.
48. 1519368 by 696.
49. 2103409 by 649.
50. 2815272 by 732.
51. 2592840 by 620.
52. 3564288 by 819.
53. 4227328 by 896.
54. 3516825 by 975.
55. 2802690 by 990.
56. 8901207 by 1449.
57. 9572160 by 1560.
58. 6192138 by 1653.
59. 3515772 by 1736.
60. 9876480 by 1976.
61. 24197460 by 2492.
62. 8231505 by 1905.
63. 13896225 by 2975.
64. 16084440 by 5058.
65. 23103465 by 6391.
66. 18356508 by 16074.
67. 576105376 by 78617.
68. 344943192 by 134376.
69. 1806147429 by 35805

CASE IV.

69. When ciphers are on the right of the divisor.

1. Divide 7654 by 500.

SOLUTION.—We find how many times 5 *hundreds* is contained in 76 *hundreds* by dividing 76 by 5. It is contained 15 times, with a remainder of 1 hundred, which, with 54, equals 154

OPERATION.

5|00)76|54

15–154

or $15\frac{154}{500}$

RULE.—1. *Cut off the ciphers at the right of the divisor, and as many places from the right of the dividend.*

2. *Divide the remaining part of the dividend by the remaining part of the divisor; prefix the remainder to the figures cut off, for the true remainder.*

NOTE.—When the divisor, with the ciphers cut off, is greater than 12, we will of course divide by long division.

Divide	Divide
2. 189 by 50.	11. 18732 by 1600.
3. 487 by 60.	12. 28732 by 1700
4. 985 by 80.	13. 19873 by 1900.
5. 1837 by 400.	14. 25307 by 2100.
6. 2572 by 1100.	15. 40302 by 2500.
7. 4783 by 1200.	16. 87316 by 3400.
8. 8725 by 1300.	17. 92913 by 4600.
9. 4687 by 1400.	18. 31200 by 5100.
10. 9876 by 1500.	19. 8732000 by 12300.

PRACTICAL PROBLEMS.

CASE I.

70. To divide a number by an equal part.

1. At 5 dollars each, how many sheep can you buy for 675 dollars?

SOLUTION.—If 5 dollars will buy one sheep, 675 dollars will buy as many sheep as 5 is contained times in 675, which are 135. Hence, *you can buy 135 sheep.*

OPERATION.

5)675

135 Ans

2. At 12 dollars each, how many pigs can you buy for 3780 dollars? Ans. 315.

3. At 6 cents apiece, how many oranges can you buy for 354 cents? Ans. 59.

4. At 11 cents a quart, how many quarts of cherries can you buy for 1243 cents? Ans. 113.

5 In one pound there are 12 ounces; how many pounds in 1728 ounces?

6. In one minute there are 60 seconds; how many minutes in 12900 seconds?

7. How many cows can you buy for 2952 dollars, at the rate of 24 dollars each?

8. How many pounds of butter will 8100 cents buy, at the rate of 25 cents a pound?

9. There are 16 ounces in one pound; how many pounds in 5472 ounces?

10. In one bushel there are 32 quarts; how many bushels are there in 16182 quarts?

11. How many acres of land at 56 dollars an acre can you buy for 12152 dollars?

12. How long will it take a vessel to sail 6460 miles, at the rate of 68 miles a day?

13. The diameter of the earth is nearly 8000 miles; how long will it take a person to walk the distance, at the rate of 48 miles a day?

14. The circumference of the earth is nearly 25000 miles; how long will it take a person to walk it, at the rate of 50 miles a day?

15. The distance to the moon is 240,000 miles; how long would it take a balloon to reach it, moving at the rate of 75 miles an hour?

16. The sun is 95,000,000 miles from the earth; how long would it require a cannon-ball to reach it, moving at the rate of 48 miles a minute?

Case II.

71. To divide a number into equal parts.

1. A man divides 387 dollars equally among 9 boys; how many dollars does each receive?

Solution.—Each boy will receive as many dollars as 9 is contained times in 387, which are 43 dollars.

Operation.

9)387

43 Ans.

2. A lady divides 4860 dollars equally among 12 girls; how many dollars will each receive? Ans. 405.

3. A man earns 2639 dollars in 13 weeks; how much does he earn in one week? Ans. 203.

4. A man travels 1728 miles in 36 days; how far does he travel each day?

5. There are 25 pounds in a quarter; how many quarters are there in 34450 pounds?

6. There are 6468 cubic inches in 28 gallons; how many cubic inches in one gallon?

7. Sound moves 37060 feet in 34 seconds; how far will it move in 48 seconds?

8. There are 2583 gallons in 41 hogsheads; how many gallons in one hogshead?

9. If a road 57 miles long cost 7695 dollars, how much did it cost a mile?

10. A man gave 1725 dollars for cows worth 25 dollars each; how many cows did he buy?

11. How many bushels of oats at 56 cents a bushel can be bought for 13272 cents?

12. A man gave 1905 dollars for saddles worth 15 dollars each; how many did he buy?

13. A farmer sold 24 horses for 5640 dollars; how much did he receive apiece for them?

14. A farmer sold a lot of horses for 7685 dollars; how many did he sell, if he received 145 dollars each?

15. How many mules can you buy for 8332 dollars, *at the rate of 184* dollars each?

MISCELLANEOUS PROBLEMS.

(Quite young pupils will omit these until review.)

PRACTICAL PROBLEMS IN ADDITION.

1. A man left 850 dollars to his daughter, and 945 dollars to each of his two sons; how much did he leave his two sons? how much did he leave all?

2. Washington was born in the year 1732, Jefferson 11 years after, and Hamilton 15 years after Jefferson; when was Jefferson born? when was Hamilton born?

3. A farmer owns three farms; the first is worth 6560 dollars, the second 385 dollars more, and the third 1387 dollars more than the second; what is the value of the second farm? of the third farm? of all?

4. A has 7586 cents, B has 596 more than A, and C has as many as A and B together; how many has B? how many has C? how many have all?

5. B walked 876 miles, C walked 285 miles more than B, and D walked 985 miles more than C; how far did C walk? how far did D walk? how far did they together walk?

6. A man gave to his wife 4675 dollars, to his son 7582 dollars, to his daughter 3594 dollars, and had 8575 dollars left; what was his fortune?

7. A owns a farm worth 3750 dollars, a wood-lot worth 856 dollars more, and a store worth 987 dollars more than both; what was the value of the wood-lot? of the store? of all three?

8. A man had two sons and three daughters; he gave each son 5896 dollars, and each daughter 4385 dollars; how much did he give to his sons? to his daughters? to all?

9. *A butcher* sold to one man 876 pounds of meat, to *nother man* 587 pounds more, and to another, 395

pounds more than both; how much did he sell to the second man? to the third man? how much to all?

10. A raised 3456 bushels of wheat, which was 2475 bushels less than B raised, and D raised 3489 bushels more than both; how much did B raise? how much did D raise? how much did all raise?

11. A bought some land for 8759 dollars, a house for 3768 dollars, and sold them so as to gain 1389 dollars; for what did he sell them?

12. A man bought two lots for 3750 dollars each; and in selling them he gained 278 dollars on the first, and 389 dollars on the second; how much did he gain on both? how much did he receive for both?

13. A has 757 acres of land, B has 285 acres more than A, and C has as many as A and B both; how many acres has B? how many has C? how many have all?

14. William lends his brother 3785 dollars, his sister 4261 dollars, and a friend 485 dollars more than his sister, and has 5858 dollars remaining; how much did he lend his friend, and what was his whole fortune?

PRACTICAL PROBLEMS
in Addition and Subtraction.

1. Find the sum of six hundred and five and 18 hundred and ninety-seven.

2. Subtract one thousand and nine from four thousand and seven.

3. Subtract $7567 + 896$ from $4875 + 4736 + 2539$.

4. A had 472 hens, and bought 589 hens, and then sold 985; how many had he then?

5. A farmer had 397 pigs, and bought 85 pigs, and then sold 182 pigs; how many had he then?

6. A drover sold his cows for 2575 dollars, and his sheep for 976 dollars, and gained 594 dollars; how much did he pay for them?

7. *A man having* 1600 acres of land sold 546 acres

to B, and 289 acres more to C than to B; how much did he sell to C? how much to both? how much remained?

8. Mr. Peters, having 4300 bushels of wheat, sold 1480 bushels, and then bought 1856 bushels more than he sold; how many bushels had he then?

9. Henry had 756 dollars, and his mother gave him enough to make his money 1200 dollars; how much did his mother give him?

10. Sarah bought 575 pins; her mother gave her 289, and her sister gave her enough to make her number 1000; how many did she receive from her sister?

11. Mary's father left her 596 acres of land; she sold 484 acres, and then bought 396 acres; how many acres had she then?

12. A sold 4760 bushels of grain, then sold 1780 bushels, and then had 1875 bushels; how many bushels had he at first?

13. B sold 7560 bushels of rye, then bought 2580 bushels, and then had 5680 bushels; how much did he sell more than he bought? how many bushels had he at first?

14. William had 456 dollars; his father gave him 2528 dollars, he then lost 1869 dollars, and gave away 286 dollars; how many dollars had he then?

15. Three men bought a farm for 20000 dollars; the first paid 7580 dollars, the second paid 6765 dollars, and the third the remainder; how much did the first and second pay? how much did the third pay?

16. A man deposited 8000 dollars in the bank; he drew out at one time 2575 dollars, at another 3467 dollars, at another 1576 dollars; how much remained in the bank?

17. Mr. Bowman, whose property was 35000 dollars, willed 9650 dollars to each of his two sons, 8750 to his daughter, and the remainder to his wife; how much did the children receive? how much did his wife receive?

PRACTICAL PROBLEMS
in Addition, Subtraction, and Multiplication.

1. What is the value of $467 \times 672 - 31675$?

2. What is the value of 672×36 plus 216×42?

3. A man sold his house for 27 times 98 dollars, plus 397 dollars; how much did he receive for it?

4. A sold 24 horses for 168 dollars each, and 63 cows for 34 dollars each; what did he receive for the horses? for the cows? for all?

5. I bought 76 oxen at 68 dollars each, and 327 sheep at 12 dollars each; how much did the oxen cost? how much the sheep? how much did all cost?

6. My barn cost 2318 dollars, my house 3 times as much, and my farm as much as both; what was the cost of the house? the cost of the farm?

7. A man bought 235 cows at 24 dollars each, and sold them for 32 dollars each; how much did he gain by the transaction?

8. A drover bought 78 horses at 164 dollars each, 215 oxen at 59 dollars each, and sold them all for 30000 dollars; how much did he gain?

9. How much must I pay for 7 building-lots, at 2348 dollars each, 5 houses, at 4250 dollars each, and 6 boats, at 3980 dollars each?

10. I bought 78 sheep at 7 dollars a head, and sold them so as to gain 267 dollars; how much did I receive for them?

11. I bought 185 acres of land at 95 dollars an acre, and in selling it I lost 2486 dollars; how much did I receive for it?

12. A speculator bought a farm of 327 acres at 79 dollars an acre, and sold it at 95 dollars an acre; how much did he gain?

13. A man sold his oil stock for 14000 dollars, and then bought a farm containing 93 acres, at 125 dollars

an acre; how much money has he left after paying for it?

14. A clerk receives a salary of 75 dollars a month; he spends 18 dollars a month for board, and 9 dollars for other expenses; how much can he save in 1 month? in 12 months?

15. A farmer having 3420 dollars bought 35 cows at 24 dollars a head, and 36 oxen at 54 dollars a head; how much has he left, after paying for them?

16. Thomas travels 24 miles a day, and Walton travels 52 miles a day; how much farther does Walton travel in 72 days than Thomas?

17. A man bought 336 bushels of potatoes at 65 cents a bushel, and 3 times as many bushels of apples at 98 cents a bushel; what was the entire cost?

18. A's barn cost 1980 dollars, his house 2150 dollars more than the barn, and his farm cost 14 times as much as the barn and house together; what was the cost of the farm?

PRACTICAL PROBLEMS

in Addition, Subtraction, Multiplication, and Division.

1. Divide 42624 by 36 and add 3146 to the quotient.

2. Divide 73305 by 45 and subtract the quotient from 8702.

3. Subtract 3125 from 5213, divide the remainder by 9, and add the quotient to 1745.

4. A man having 18000 dollars leaves his wife 4800 and divides the remainder equally among 6 children; what does each receive?

5. A farm of 24 acres was bought for 4056 dollars and sold at a gain of 3168 dollars; for what was it sold per acre?

6. If 27 men share 11286 dollars equally, how much would each have? how much would A have, if he had four times as much as each, plus 1245 dollars?

7. If 29 men earn 7946 cents in a day, and 25 boys earn 5450 cents in a day, how much more does one man earn in a day than one boy?

8. A horse and 18 oxen are worth 1001 dollars; now, if the horse is worth 245 dollars, what is the value of the oxen? of each ox? Ans. 42 dollars.

9. The value of 3 horses and 15 cows is 1155 dollars; if the value of each horse is 225 dollars, what is the value of each cow? Ans. 32 dollars.

10. If you divide 60466 by 49, by what number must I multiply the quotient to produce 9872? Ans. 8.

11. The income of a man who "struck oil" is 400 dollars per day; how many teachers would this employ at a salary of 730 dollars a year?

12. I bought 326 barrels of flour for 2608 dollars, paid 46 dollars for transportation, and sold it at a gain of 280 dollars; what did I receive a barrel?

Ans. 9 dollars.

13. I sold a farm containing 190 acres for 65 dollars an acre, and bought with the proceeds another farm at 95 dollars an acre; how many acres in the latter farm?

14. A drover bought 234 cows at 25 dollars each, and sold 95 of them at cost each; how much must he receive a head for the remainder, to gain 973 dollars?

Ans. 32.

15. If the President of the United States expends 104 dollars daily, how much can he save in a year of 365 days, out of his salary of 50000 dollars?

16. If the Vice-President expends 35 dollars daily, how much can he save at the end of the year, if he has an income of 6450 dollars, besides his salary of 8000 dollars a year?

17. If the Secretary of State expends 16 dollars a day, how much can he save in a year, his salary being 8000 dollars a year and his private income 28 dollars a week?

SECTION IV.

UNITED STATES MONEY.

72. ***United States Money*** is the money of the United States.

TABLE.

10 mills (*m.*) equal	. . . .	1 cent, *c.*
10 cents	"	1 dime, *d.*
10 dimes	"	1 dollar, $.
10 dollars	"	1 eagle, E.

Coins are pieces of metal, stamped by the authority of the government, to be used as money.

GOLD COINS.			SILVER COINS.		
Eagle,	*value*	*$10*	*Dollar,*	*value*	*$1*
Double-eagle,	"	*20*	*Half-dollar,*	"	*50 c.*
Half-eagle,	"	*5*	*Quarter-dollar,*	"	*25 c.*
Quarter-eagle,	"	*2½*	*Dime,*	"	*10 c.*
Dollar,	"	*1*	*Half-dime,*	"	*5 c.*
Three-dollar,	"	*3*	*Three-cent,*	"	*3 c.*

NICKEL.			BRONZE.		
Three-cent,	*value*	*3 c.*	*Two-cent,*	*value*	*2 c.*
Five-cent,	"	*5 c.*	*Cent,*	"	*1 c.*

NUMERATION AND NOTATION.

73. The *dollar* is indicated by the symbol $. The eagle and dollar are read as a number of dollars: thus, 3 eagles and 5 dollars are read, 35 dollars.

74. The *dime* is one-tenth of a dollar, and is written to the right of the dollar and separated from it by a point, called a separatrix; thus, $3.4 represents 3 dollars and 4 dimes.

75. The *cent* is 1 tenth of a dime, or 1 hundredth of a dollar. It is written two places to the right of dollars; thus, *$4.58 represents* 4 dollars, 5 dimes, and 8 cents.

76. Dimes and cents are usually read as so many cents; thus, $7.45 is read, 7 dollars and 45 cents.

77. The *mill* is 1 tenth of a cent, and is written one place to the right of cents; thus, $5.475 is read, 5 dollars, 47 cents, and 5 mills.

PRACTICAL PROBLEMS.

EXAMPLES IN NUMERATION.

1. Write and read $24.75.

SOLUTION.—The pupil will write this upon the slate or black board, and say: This is read, 24 dollars, 7 dimes, and 5 cents; or, 24 dollars and 75 cents.

The pupil will write and read the following:

2. $14.25	6. $48.50	10. $105.076
3. $24.67	7. $50.06	11. $976.705
4. $19.84	8. $48.408	12. $350.035
5. $28.574	9. $96.004	13. $847.008

EXAMPLES IN NOTATION.

1. Write six dollars and twenty-five cents.

2. Write twenty-five dollars and thirty-six cents.

3. Write eight dollars, forty-five cents, and six mills.

4. Write twenty dollars, seventy-five cents, and two mills.

5. Write six eagles, seven dollars, and eighty-four cents.

6. Write four dollars, six dimes, and seven cents.

7. Write 25 dollars, five cents, and eight mills.

REDUCTION OF UNITED STATES MONEY.

78. ***Reduction*** consists in changing the denomination without changing the value. From the table we derive the following principles:

To reduce cents to mills, we multiply the cents by 10, or annex ONE cipher.

To reduce dollars to cents, we annex TWO *ciphers.*

To reduce dollars to mills, we annex THREE *ciphers.*

To reduce a number of dollars and cents to cents, we remove the decimal point; thus, $5.24 = 524 *cents.*

CASE I.

To reduce to lower terms.

1. Reduce 6 dollars to cents.

SOLUTION.—In 1 dollar there are 100 cents; hence, in 6 dollars there are 6 times 100 cents, or 600 cents; or we annex two ciphers.

OPERATION.
$6 = 600 cents

2. Reduce $18 to cents.
3. Reduce $24 to cents.
4. Reduce $385 to cents.
5. Reduce $27 to mills.
6. Reduce 85 cents to mills.
7. Reduce $5.47 to cents.
8. Reduce $27.05 to cents.
9. Change $9 607 to mills.

CASE II.

To reduce to higher terms.

79. From the table we have the following principles:

1. *To reduce cents to dollars, place the point* TWO *places from the right.*

2. *To reduce mills to dollars, place the point* THREE *places from the right.*

1. Reduce 2347 cents to dollars.

SOLUTION.—There are 100 cents in 1 dollar, and in 2347 cents there are as many dollars as 100 is contained times in 2347, which are $23.47; or we place the point two places from the right.

OPERATION.
$2347 \div 100$
$= \$23.47$

2. Reduce 845 cents to dollars. Ans. $8.45.
3. Reduce 2835 cents to dollars. Ans. $28.35.

4. Reduce 46785 cents to dollars.
5. Reduce 7895 mills to dollars.
6. Reduce 27065 mills to dollars.
7. Reduce 4800 cents to dollars.
8. Reduce 9600 mills to dollars.

ADDITION OF UNITED STATES MONEY.

80. ***Addition of United States Money*** is performed as in simple numbers, according to the following

RULE.—1. *Write dollars under dollars, cents under cents, etc.*

2. *Add as in simple numbers, and place the separatrix between dollars and cents.*

1. Find the sum of $24.36, $96.58, and $75.42.

SOLUTION.—We write dollars under dollars and cents under cents, and commence at the right to add. 2 and 8 are 10, and 6 are 16 cents; which equals 6 cents and 1 dime; we write the 6 cents under the column of cents, and add the 1 dime to the next column, etc.

OPERATION.

$24.36
96.58
75.42
$196.36

2. Find the sum of $48.56, $39.46, $24.67, and $81.09.
3. Add $23.84, $97.36, $52.75, and $98.27.
4. Add $73.75, $48.56, $39.87, and $75.48.
5. Add $46.375, $97.283, $72.475, and $8.396.
6. Add $156.96, $284.076, $9.27, and $85.735.
7. A man bought a cow for $24.75, a horse for $150.50, a wagon for $287.75, and a carriage for $375.87; how much did he pay for all?
8. A merchant bought flour for $57.35, some calico for $96.87, some cloth for $84.50, some boots for $52.87, and some muslin for $75.75; what did they all cost?
9. A tailor sold a coat for $34.75, a vest for $8.50, a cloak for $52.25, a pair of pants for $9.75, and some other things for $28.45; what did he receive for all?
10. I bought a table for $18.25, a looking-glass for

$25.75, a bedstead for $36.50, a bureau for $46.25; what did they all cost?

11. A owes $624.30, B owes $467.56, C owes $359.45, D owes $95.12, E owes $43.84, F owes $27.75, G owes $968.47, H owes $7.75; required the sum of their debts.

SUBTRACTION OF UNITED STATES MONEY.

81. ***Subtraction of United States Money*** is performed as in subtraction of simple numbers, according to the following

RULE.—1. *Write dollars under dollars, cents under cents, etc.*

2. *Subtract as in simple numbers, and place the separatrix between dollars and cents.*

1. Subtract $21.48 from $46.73.

SOLUTION.—We cannot subtract 8 cents from 3 cents, hence we add 10 cents to 3 cents, making 13 cents; 8 cents from 13 cents leave 5 cents. Now, since we added 10 cents, or 1 dime, to the minuend, we must add 1 dime to the 4 dimes, making 5 dimes: 5 dimes from 7 dimes leave 2 dimes, etc.

OPERATION.

$46.73	
27.48	
$19.25	

(2.)	(3.)	(4.)	(5.)
$78.25	$57.52	$96.43	$75.75
13.16	23.28	28.14	23.28

6. From $129.39 take $48.91.

7. Find the difference between $234.16 and $471.24.

8. A man bought a horse for $234.50, and sold it for $228.25; what did he lose?

9. A merchant bought cloth for $96.75, and sold it for $110.29; what did he gain?

10. A bought a farm for $3640.25, and sold it for $4000; what was the gain?

11. My house cost $3480.75, and I sold it for $4000.50; what did I gain?

12. My horse cost $240.50, and my carriage cost $386.25; I sold them for $680.50; what did I gain?

13. A merchant bought cloth for $325.50, muslin for $436.75, and flour for $625.80; he sold them all for $1300: how much did he lose?

14. I paid $4637.25 for a farm, paid $3675.25 for building a house, and $2896.87 for building a barn; I sold my property for $13000; how much did I gain?

15. I paid $246.75 for a horse, $325.45 for a mule, $42.25 for an ox, $37.50 for a cow; I sold them all for $603.50; what was the loss?

MULTIPLICATION OF UNITED STATES MONEY.

82. ***Multiplication of United States Money*** is performed, like multiplication of simple numbers, according to the following

RULE.—*Multiply as in simple numbers and place the separatrix between dollars and cents.*

1. Multiply $36.25 by 3.

SOLUTION.—Three times 5 cents are 15 cents, which equal 1 dime and 5 cents; we write the 5 cents, and reserve the 1 dime to add to the next product. 3 times 2 dimes are 6 dimes, and 6 dimes plus 1 dime are 7 dimes, etc.

OPERATION.

$36.25
3
$108.75

Multiply	Multiply
2. $26.14 by 4.	7. $48.25 by 12.
3 $37.27 by 5.	8. $72.27 by 13.
4 $48.96 by 7.	9. $85.58 by 15.
5. $37.52 by 8.	10. $92.83 by 32.
6. $79.35 by 9.	11. $75.32 by 46.

12. If one yard of cloth cost $3.25, what cost 5 yards?

13. What will 12 horses cost at the rate of $150.75 a piece?

14. A man bought 27 oxen at the rate of $36.25 each, what did they cost?

15. A farmer sold 325 bushels of wheat at $1 25 a bushel; how much did he receive for it?

16. A miller sold 472 barrels of flour at $7.87 a barrel; how much did he receive for it?

17. A man bought 47 cows for $24.30 each, and sold them for $28.10 each; what was the gain?

18. A drover bought 247 horses for $130.75 each, and sold them for $180.30 each; what did he gain?

19. A farmer bought 327 acres of land at $76.25 an acre, and sold it at $92.50 an acre; what did he gain?

DIVISION OF UNITED STATES MONEY.

83. ***Division of United States Money*** is performed like division of simple numbers.

CASE I.

84. To divide a number into equal parts.

RULE.—*Divide as in simple numbers, and place the separatrix between dollars and cents.*

1. Divide $7.32 in 3 equal parts, or find 1 third of it.

SOLUTION.—1 third of 7 dollars is 2 dollars, and 1 dollar remaining; 1 dollar equals 10 dimes, which, added to 3 dimes, equal 13 dimes. 1 third of 13 dimes equals 4 dimes, and 1 dime remaining, etc.

OPERATION.

3)$7.32

$2.44 Ans.

2. Divide $9.24 into 4 equal parts.
3. Divide $7.25 into 5 equal parts.
4. Divide $17.22 into 6 equal parts.
5. If 7 pigs cost $36.75, what will one pig cost?
6. If 8 cows cost $172.80, what will one cow cost?
7. If 3 oxen cost $325.20, what will 5 oxen cost?
8. If 7 hens cost $3.15, what will 12 hens cost?
9. What cost 15 sheep, if 4 sheep cost $29.24?
10. What cost 25 pounds of butter, if 7 pounds cost *2.38?*

11. What cost 34 acres of land, if 12 acres cost $5.04?
12. What cost 28 cows, if 35 cows cost 987 dollars?
13. What cost 75 oxen, if 38 oxen cost 1615 dollars?
14. What cost 234 hens, if 75 hens cost $25.50?

Case II.

85. To divide one sum of money by another.

Rule.—*Reduce both sums to the same denomination, and divide as in simple numbers.*

1. Divide $736 by $4.

Solution.—Dividing as in simple numbers, we have 184.

OPERATION.

4)736

184 Ans.

2. Divide $9600 by $16.
3. Divide 728 cents by 4 cents.
4. Divide 3625 cents by 5 cents.
5. Divide $26325 by 81 dollars.
6. At 24 dollars each, how many cows can you buy for 1344 dollars?
7. At 42 dollars each, how many oxen can be bought for $3276?
8. At $3.25 apiece, how many pigs can you buy for $120.25?
9. A earned $3.75 a day; how many days did he work to earn $78.75?
10. A drover paid $6972 for horses, at $145.25 apiece; how many did he buy?
11. How many cords of wood can you buy for $312, at $3.25 a cord?
12. William earned $3.25 a day, and paid 75 cents for board; in how many days would he save $912.50?
13. A merchant received $853.25 for a case of silk, including $1.25 cost of box. How many pieces of silk were in the case, if he received $53.25 apiece?

BILLS AND ACCOUNTS.

86. A ***Bill of Goods*** is a written statement of goods bought or sold, giving the place, date, names of buyer and seller, quantity, price, and entire cost.

An ***Account*** is a written statement of the debts and the credits of business transactions.

The party who owes is the *debtor;* the party who is owed is the *creditor.* A bill is made out by the following

RULE.—1. *Find the cost of the several items by multiplying the price of each by the quantity, and take the sum of the several products.*

2. *In an* ACCOUNT, *find the difference between the debit and credit amounts.*

Make out the following bills:

(1.) *Millersville, May 8, 1864.*

Mr. Harry Bowman,

Bought of HENRY MARTIN,

8	*yds. of muslin, at $0.27,*	$
12	*" of cloth, " 2.37,*	
15	*" of silk, " 1.62,*	
	Amount due,	$

(2.) *Lancaster, April 6, 1864.*

Theo. Miller,

Bought of DANIEL MOONEY,

24	*pairs boots, at $5.25,*	$
37	*" gaiters, " 3.75,*	
45	*" slippers, " 1.37,*	
28	*" rubbers, " 1.25,*	
	Amount due,	$

Received Payment,

Daniel Mooney.

(3.) *New York, Dec. 17, 1862.*

John J. Brooks,

Bought of CHARLES HOYT,

47	*bbls. of flour, at $7.35,*	$	
28	*lbs. of beef, " 0.37,*		
97	*yds. of cloth, " 2.75,*		
146	*bu. of wheat, " 1.12,*		
	Amount,	$	

Received Payment,

Charles Hoyt.

(4.)

John Smith, Dr.

1866.					
Jan.	*1*	*To 75 lbs. of sugar, at $0.35,*	$		
Feb.	*5*	*" 47 yds. of cloth, " 3.25,*			
		Cr.			
Jan.	*7*	*By 75 bu. of corn, at $0.78,*		$	
Feb.	*2*	*" 83 bu. of apples, " 1.25,*			
		Balance due,			

(5.) *Philadelphia, April 1, 1860.*

Mr. Henry Farnam, Dr.

To Edwin Lamborn.

1860.					
Jan.	*4.*	*To 145 bu. wheat, at $1.25,*	$		
Jan.	*10.*	*" 236 " rye, " 1.05,*			
Jan.	*20.*	*" 176 " oats, " 0.65,*			
1860.		*Cr.*			
Jan.	*3.*	*By 45 yds. cloth, at $3.65,*		$	
Jan.	*12.*	*" 72 " silk, " 2.12,*			
Feb.	*24.*	*" 80 " cassimere, " 1.75,*			
		Balance due,			

Received Payment,

Edwin Lamborn.

COMPOSITION AND FACTORING.

87. A ***Composite Number*** is one that can be produced by multiplying two or more numbers together, each of which is greater than a unit.

Thus, 6 is a composite number, since it can be produced by multiplying 3 and 2 together, each of which is greater than a unit.

A ***Prime Number*** is one that cannot be produced by multiplying two or more numbers together, each greater than a unit.

Thus, 2, 3, 5 and 7 are prime numbers, since they cannot be produced by multiplying together any two numbers, each greater than a unit.

The ***Factors*** of a composite number are the numbers which, when multiplied together, will produce it.

Thus, 2 and 3 are the factors of 6, since 3 times 2 are 6; 4 and 3 are the factors of 12, since 4 times 3 are 12.

The ***Prime Factors*** of a number are the prime numbers which, when multiplied together, will produce it.

Thus, 2, 2 and 3 are the prime factors of 12.

MENTAL EXERCISES.

1. What numbers multiplied together will produce 6, 10, 12, 14, 15, 18, 20, 24, 33, 72, 84, 108, 156?

2. What are the factors of 10, 14, 15, 18, 21, 24, 25, 27, 28, 32, 33, 42, 55, 72, 96, 144, 216?

What prime numbers multiplied together will produce 6, 8, 12, 15, 16, 18, 20, 22, 24, 28, 35, 40, 56, 74, 125, 186?

4. What are the prime factors of 12, 18, 27, 36, 40, 64, 96, 132?

5. Tell which of the following numbers are prime, and which composite: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25.

COMPOSITION.

88. ***Composition*** is the process of composing numbers out of their factors.

Thus, the production of 12 out of its factors, 3 and 4, is composition.

PRINCIPLE.—*Every composite number is equal to the product of its prime factors.*

1. Find the composite number whose factors are 2, 3 and 5.

SOLUTION.—To find the composite number whose factors are 2, 3 and 5, find the product of these factors. 5 multiplied by 3 is 15, and 15 multiplied by 2 is 30. Hence the composite number is 30.

OPERATION.

$$\begin{array}{r} 5 \\ 3 \\ \hline 15 \\ 2 \\ \hline 30 \end{array}$$

RULE.—*Find the product of all the factors.*

2. Find the composite number composed of the factors 3, 5 and 7. Ans. 105.

3. Find the composite number composed of the factors 7, 9, 17 and 89. Ans. 95319.

4. Find the composite numbers consisting of three equal factors, each being 5; each 7; each 15; each 59.

5. Find the composite numbers which have three equal factors, when each is 35; 87; 109; 163; 530; 657.

6. Find the composite numbers consisting of four equal factors, each being 3; 11; 17; 44; 54; 75; 153.

7. Find the composite number produced by the five smallest primes, including 1; by the nine smallest composite numbers. Ans. 1st, 210. 2d, 696729600.

FACTORING.

89. ***Factoring*** is the process of finding the factors of composite numbers.

Thus, the finding of the factors, 3 and 4, of 12, is factoring.

PRINCIPLE.—*Every composite number is divisible by its prime factors.*

Thus, 15 is the product of its two prime factors, 3 and 5; hence 15 is divisible by 3 or 5.

1. What are the prime factors of 60?

SOLUTION.—Dividing 60 by 2 we have a quotient of 30; dividing 30 by 2 we have a quotient of 15; dividing 15 by 3 we have a quotient of 5; hence 2, 2, 3 and 5 are the factors of 60, and since they are prime numbers they are the prime factors of 60.

OPERATION.

$$\begin{array}{r} 2)60 \\ 2)30 \\ 3)15 \\ \hline 5 \end{array}$$

RULE.—*Divide the number by any prime number, except* 1, *that will exactly divide it; divide the quotient, if composite, in the same manner, and thus continue until the quotient is a prime number. The last quotient and the divisors are the prime factors required.*

Find the prime factors of

2. 48	6. 175	10. 475	14. 1200
3. 72	7. 270	11. 858	15. 7290
4. 81	8. 315	12. 1575	16. 29295
5. 108	9. 336	13. 8316	17. 341775

GREATEST COMMON DIVISOR.

90. A ***Divisor*** of a number is a number that will exactly divide it.

Thus, 4 is a divisor of 12, since it divides 12 without a remainder.

A ***Common Divisor*** of two or more numbers is a number that will exactly divide each of them.

Thus, 4 is a common divisor of 16 and 24, since it divides each of them without a remainder.

The ***Greatest Common Divisor*** of two or more numbers is the greatest number that will exactly divide each of them.

Thus, 18 is the greatest common divisor of 36 and 54, since it is the greatest number that will divide each of them without a remainder.

MENTAL EXERCISES.—1. Name some divisors of 8; 12; 18; 24; 36.

2. What factors are common to 8 and 12? 9 and 12? 20 and 30?

3. What divisors are common to 12 and 16? 18 and 24? 36 and 48?

4. What is the largest divisor common to 8 and 12? to 12 and 14? to 12 and 16? to 24, 36 and 72? to 25, 50 and 125?

PRINCIPLE.—*The greatest common divisor of two or more numbers equals the product of all the common prime factors of those numbers.*

1. Find the greatest common divisor of 24, 30 and 42.

SOLUTION.—The factors of 24 are 2, 3 and 4; the factors of 30 are 2, 3 and 5; the factors of 42 are 2, 3 and 7. The common factors of 24, 30 and 42 are *2 and 3;* and the product of 2 and 3, or 6, is the *greatest* common divisor of 24, 30 and 42.

OPERATION.

$24 = 2 \times 3 \times 4$

$30 = 2 \times 3 \times 5$

$42 = 2 \times 3 \times 7$

$2 \times 3 = 6$

RULE.—*Resolve the numbers into their prime factors and take the product of all the common prime factors.*

Find the greatest common divisor

2. Of 30 and 36. Ans. 6.
3. Of 60 and 90. Ans. 30.
4. Of 44 and 66. Ans. 22.
5. Of 96 and 84. Ans. 12.
6. Of 175 and 245. Ans. 35.
7. Of 12, 15 and 21. A. 3.
8. Of 18, 24 and 36. A. 6.
9. Of 36, 72 and 108. A. 36.
10. Of 84, 126 and 210. A. 42.
11. Of 556, 630 and 1638.

12. What is the length of the longest pole with which you can measure 126, 144 and 156 feet? Ans. 6 feet.

13. Three pieces of carpet, of 48, 64 and 80 yards, will exactly cover Mrs. White's parlor, if cut into the longest possible equal lengths. How long is the parlor? and how wide, if breadth of carpet is one yard?

Ans. Length 16 yards; Width, 12 yards.

LEAST COMMON MULTIPLE.

91. A ***Multiple*** of a number is one or more times that number.

Thus, 12 is a multiple of 4, since it is *three* times 4.

A ***Common Multiple*** of two or more numbers is a number which is a multiple of each of them.

Thus, 24 is a common multiple of 4 and 6, since it is a number of times each of them.

The ***Least Common Multiple*** of two or more numbers is the least number which is a multiple of each of them.

Thus, 12 is the least common multiple of 4 and 6, since it is the least number that is a number of times each of them.

MENTAL EXERCISES.—1. What number is a multiple of 3? of 4? of 5? of 6? of 7? of 8?

2. Name two multiples of 8; two multiples of 10; three multiples of 9; three multiples of 12.

3. What number is a multiple of both 4 and 6? 5 and 6? 6 and 8?

4. Name a common multiple of 3 and 4; 6 and 9; 8 and 12; 9 and 12.

5. Name the least common multiple of 4 and 6; of 4 and 8; of 6 *and 8; of 8 and 10;* of 9 and 12.

PRINCIPLE.—*The least common multiple of two or more numbers must contain all the factors of each number, and no other factors.*

1. Find the least common multiple of 6 and 15.

SOLUTION.—The prime factors of 6 are 2 and 3; hence the multiple must contain the factors 2 and 3. The factors of 15 are 3 and 5; hence the multiple must contain the additional factor, 5. The least common multiple, therefore, of 6 and 15 is $2 \times 3 \times 5$, or 30.

OPERATION.

$6 = 2 \times 3$

$15 = 3 \times 6$

L. C. M. $= 2 \times 3 \times 5 = 30$.

RULE.—*Resolve the numbers into their prime factors, and take the product of all the different factors, using each factor the greatest number of times it occurs in either number.*

NOTE.—If one number is a divisor of another, omit it.

Find the least common multiple

2. Of 12 and 15. Ans. 60.
3. Of 15 and 18. Ans. 90.
4. Of 16 and 18. Ans. 144.
5. Of 48 and 72. Ans. 144.
6. Of 27 and 135. Ans. 135.
7. Of 6, 8 and 10. A. 120.
8. Of 5, 9, 12 and 15. A. 180.
9. Of 12, 15, 18 and 24.
10. Of 20, 84, 96 and 108.
11. Of 63, 105, 189 and 294.

12. At a sunday-school collection, four classes contributed equal amounts. In one class each member gave 5 cents; in another, 6 cents; in another, 8 cents; and in the fourth, 10 cents: what is the least sum with which this could happen? Ans. $1.20.

13. The piece-goods in a case of silk are to cut without waste into dress patterns of either 12, 15, 20 or 30 yards: what are the shortest lengths into which the piece-goods can be made? Ans. 60 yards.

CANCELLATION.

92. ***Cancellation*** is a process of shortening computations by rejecting common factors from the dividend and divisor.

PRINCIPLE.—*Cancelling a common factor from both dividend and divisor does not change the quotient.*

For if we divide 18 by 6, the quotient is 3; and also, if we resolve 18 and 6 into their factors, and cancel the common factor, 3, the quotient is then 3.

$$\frac{18}{6} = 3$$

$$\frac{3 \times 6}{2 \times 3} = 3$$

1. Divide 28 by 8.

SOLUTION.—Write the divisor, 8, under the dividend, 24. Resolve 28 into the factors, 4×7, and 8 into 2×4, and cancel the common factor, 4, in dividend and divisor, and we have 7 divided by 2, or $3\frac{1}{2}$

OPERATION.

$$\frac{28}{8} = \frac{4 \times 7}{2 \times 4} = \frac{7}{2} = 3\frac{1}{2}$$

RULE.—*Cancel the factors common to the dividend and divisor; then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

NOTE.—When a factor is cancelled, the unit, 1, takes its place, but need not be written, except in the quotient where there are no other factors.

2. Divide 48 by 30. A. $1\frac{3}{5}$.
3. Divide 54 by 45. A. $1\frac{1}{5}$.
4. Divide 72 by 63. A. $1\frac{1}{7}$.
5. Divide 42 by 30. A. $1\frac{2}{5}$.
6. Divide 90 by 50. A. $1\frac{4}{5}$.
7. Divide 144 by 120. A. $1\frac{1}{5}$.
8. Divide $4 \times 5 \times 6$ by 60. Ans. 2.
9. Divide 70 by $2 \times 4 \times 5$. Ans. $1\frac{3}{4}$.
10. Divide $4 \times 6 \times 8$ by $3 \times 5 \times 7$. Ans. $1\frac{29}{35}$.
11. Divide $7 \times 9 \times 10$ by $3 \times 5 \times 7$. Ans. 6.
12. Divide $8 \times 10 \times 12$ by $4 \times 5 \times 16$. Ans. 3.
13. Divide $27 \times 12 \times 14$ by $9 \times 4 \times 7$. Ans. 18.
14. Divide $72 \times 45 \times 140$ by $18 \times 24 \times 35$. A. 30.

15. How many apples, at 2 cents each, can be got for 12 oranges, at 3 cents each? Ans. 18.

16. How many pigs, at 5 dollars each, can be obtained for 20 barrels of corn, at 3 dollars a barrel? Ans. 12.

17. How many bushels of oats, worth 55 cents a bushel can be exchanged for 44 bushels of rye, at 75 cents a bushel. Ans. 60.

18. A merchant exchanged 4 pieces of gros grain silk, each containing 50 yards, at 6 dollars a yard, for beaver cloth, worth 5 dollars a yard; how many pieces, *each containing* 30 yards, did he obtain? Ans. 8.

SECTION V.

COMMON FRACTIONS.

93. A ***Fraction*** is a number of equal parts of a unit; as one half, two thirds, etc.

94. A fraction is expressed by figures with a line between; thus, $\frac{2}{3}$ expresses 2 *thirds*.

95. The number denoted by the figure below the line is called the *denominator;* it shows the number of equal parts into which the unit is divided.

96. The number denoted by the figure above the line is the *numerator;* it shows the number of equal parts considered.

97. A ***Proper Fraction*** is one whose value is less than a unit; as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{7}$, etc.

98. An ***Improper Fraction*** is one whose value is equal to or greater than a unit; as $\frac{4}{4}$, $\frac{8}{5}$, $\frac{21}{8}$, etc.

99. A ***Compound Fraction*** is a fraction of a fraction; as $\frac{1}{2}$ of $\frac{2}{3}$.

100. A ***Mixed Number*** consists of a whole number and a fraction; as $2\frac{1}{2}$, $5\frac{2}{3}$, etc.

To TEACHERS.—Give pupils a clear idea of a fraction by dividing some object, as an *apple*, by *lines* upon the blackboard, etc. For Mental Exercises, see Primary Mental Arithmetic.

MENTAL EXERCISES.

1. What is one-half?

Ans. One-half of any thing is one of the two equal parts of it

What is

2. One-third?
3. One-fourth?
4. *One-fifth?*
5. *One-sixth?*

What is

6. One-seventh?
7. One-eighth?
8. One-tenth?
9. One-twelfth?

1. What is two-thirds?

Ans. Two-thirds of any thing is two of the three equal parts of it.

What is	What is
2. Two-fourths?	6. Four-fifths?
3. Three-fourths?	7. Two-sixths?
4. Two-fifths?	8. Three-sevenths?
5. Three-fifths?	9. Four-ninths?

1. What is $\frac{1}{2}$ of 6?

Ans. $\frac{1}{2}$ of 6 is 3, since 2 times 3 are 6.

2. Find $\frac{1}{2}$ of 8.	5. Find $\frac{2}{3}$ of 15.
3. Find $\frac{1}{3}$ of 12.	6. Find $\frac{3}{4}$ of 20.
4. Find $\frac{1}{4}$ of 16.	7. Find $\frac{4}{5}$ of 30.

NUMERATION AND NOTATION.

Read the following fractions.

1. $\frac{5}{8}$; $\frac{6}{7}$.	3. $\frac{3}{8}$; $\frac{6}{11}$.	5. $\frac{14}{21}$; $\frac{7}{15}$.
2. $\frac{7}{9}$; $\frac{3}{5}$.	4. $\frac{11}{13}$; $\frac{8}{10}$.	6. $\frac{5}{7}$; $\frac{115}{9}$.

Write the following fractions.

1. Two-thirds.	4. Eight-tenths.
2. Four-fifths.	5. Seven-ninths.
3. Five-sevenths.	6. Eleven-fifteenths.

1. Analyze the fraction $\frac{3}{4}$.

SOLUTION.—In the fraction $\frac{3}{4}$, the denominator, 4, shows that the unit is divided into 4 equal parts, and the numerator, 3, shows that 3 of these parts are taken.

Analyze the following:

2. $\frac{2}{3}$; $\frac{4}{7}$.	5. $\frac{4}{5}$; $\frac{3}{11}$.	8. $\frac{9}{15}$; $\frac{7}{18}$.
3. $\frac{5}{6}$; $\frac{4}{5}$.	6. $\frac{7}{9}$; $\frac{8}{11}$.	9. $\frac{13}{21}$; $\frac{18}{23}$.
4. $\frac{3}{7}$; $\frac{2}{11}$.	7. $\frac{12}{13}$; $\frac{8}{14}$.	10. $\frac{21}{31}$; $\frac{34}{44}$.

PRINCIPLES OF FRACTIONS.

100½. We will now solve a number of problems, and derive some of the principles of fractions.

1. *Multiply* the numerator of $\frac{3}{5}$ by 2.

SOLUTION.—Multiplying the numerator of $\frac{3}{5}$ by 2, we have 6 *fifths*, which is 2 times as great as 3 *fifths*. Hence the following

OPERATION $\frac{3}{5} \times 2 = \frac{6}{5}$

PRINCIPLE I.—*Multiplying the numerator of a fraction by any number multiplies the fraction by that number.*

Multiply the fraction		Multiply the fraction
2. $\frac{3}{4}$ by 5.	Ans. $\frac{15}{4}$.	6. $\frac{13}{15}$ by 14.
3. $\frac{5}{6}$ by 7.		7. $\frac{17}{19}$ by 18.
4. $\frac{12}{15}$ by 8.		8. $\frac{18}{23}$ by 17.
5. $\frac{13}{17}$ by 11.		9. $\frac{19}{21}$ by 20.

1. Divide the numerator of $\frac{4}{5}$ by 2.

SOLUTION.—Dividing the numerator of $\frac{4}{5}$ by 2, we have 2 *fifths*, which is 1 half of 4 *fifths*. Hence the following

OPERATION. $\frac{4}{5} \div 2 = \frac{2}{5}$

PRINCIPLE II.—*Dividing the numerator of a fraction by any number divides the fraction by that number.*

Divide the fraction		Divide the fraction
2. $\frac{6}{7}$ by 3.	Ans. $\frac{2}{7}$.	6. $\frac{12}{13}$ by 4.
3. $\frac{8}{9}$ by 4.		7. $\frac{18}{26}$ by 9.
4. $\frac{10}{11}$ by 5.		8. $\frac{144}{151}$ by 12.
5. $\frac{14}{17}$ by 7.		9. $\frac{256}{321}$ by 32.

1. Multiply the denominator of $\frac{3}{4}$ by 2.

SOLUTION.—Multiplying the denominator by 2, we have 3 *eighths*, which is one-half as much as 3 *fourths*, since *eighths* are only *half* as large as *fourths*. Hence the following

OPERATION. $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

PRINCIPLE III.—*Multiplying the denominator of a fraction by any number divides the fraction by that number.*

Divide the fraction		Divide the fraction
2. $\frac{2}{3}$ by 4.	Ans. $\frac{2}{12}$.	7. $\frac{15}{22}$ by 8
3. $\frac{11}{12}$ by 7.	Ans. $\frac{11}{84}$.	8. $\frac{5}{8}$ by 6.
4. $\frac{3}{4}$ by 5.		9. $\frac{13}{16}$ by 12.
5. $\frac{18}{19}$ by 7.		10. $\frac{9}{10}$ by 11.
6. $\frac{7}{8}$ by 3.		11. $\frac{15}{19}$ by 13.

1. Divide the denominator of $\frac{3}{4}$ by 2.

SOLUTION.—Dividing the denominator by 2, we have 3 halves, and 3 halves is twice as great as 3 fourths, since *halves* are *twice* as large as *fourths*. Hence the following

PRINCIPLE IV.—*Dividing the denominator of a fraction by any number multiplies the fraction by that number.*

Multiply, by dividing the denominator,

2. $\frac{3}{8}$ by 2.
3. $\frac{3}{12}$ by 6.
4. $\frac{5}{6}$ by 3.
5. $\frac{7}{18}$ by 9.
6. $\frac{7}{8}$ by 4.
7. $\frac{19}{21}$ by 7.
8. $\frac{9}{10}$ by 5.
9. $\frac{29}{48}$ by 12.
10. $\frac{5}{18}$ by 6.
11. $\frac{41}{76}$ by 19.
12. $\frac{11}{20}$ by 10.
13. $\frac{91}{144}$ by 36.

1. Multiply both numerator and denominator of $\frac{3}{8}$ by 2.

SOLUTION.—Multiplying both numerator and denominator by 2, we have $\frac{6}{16}$; and this equals $\frac{3}{8}$, since we both multiplied and divided $\frac{3}{8}$ by 2, and hence did not change its value. Hence we have the following

OPERATION. $\frac{3}{8} \times \frac{2}{2} = \frac{6}{16}$

PRINCIPLE V.—*Multiplying both numerator and denominator of a fraction by the same number does not change the value of the fraction.*

2 Multiply both numerator and denominator of $\frac{3}{4}$ by 3; $\frac{4}{5}$ by 6; $\frac{3}{7}$ by 5; $\frac{8}{9}$ by 3; $\frac{11}{12}$ by 9; $\frac{13}{14}$ by 6.

1. Divide both numerator and denominator of $\frac{4}{6}$ by 2.

SOLUTION.—Dividing both numerator and denominator by 2, we have $\frac{2}{3}$; and this equals $\frac{4}{6}$, since we both divided and multiplied $\frac{4}{6}$ by 2, and hence did not change its value. From this we have

OPERATION. $\frac{4}{6} \div \frac{2}{2} = \frac{2}{3}$

PRINCIPLE VI.—*Dividing both numerator and denominator by the same number does not change the value of the fraction.*

2. Divide both numerator and denominator of $\frac{6}{8}$ by 2; $\frac{8}{12}$ by 4; $\frac{9}{12}$ by 3; $\frac{10}{12}$ by 2; $\frac{16}{20}$ by 4; $\frac{20}{30}$ by 10; $\frac{30}{45}$ by 15

REDUCTION OF FRACTIONS.

101. Reduction of fractions is the process of changing their form without changing their value.

CASE I.

102. To reduce mixed numbers to fractions.

1. How many thirds in $4\frac{2}{3}$?

SOLUTION.—In 1 there are $\frac{3}{3}$, and in 4 there are 4 times $\frac{3}{3}$, which are $\frac{12}{3}$, and $\frac{12}{3} + \frac{2}{3}$ equal $\frac{14}{3}$. From this solution we have the following

OPERATION.

$4\frac{2}{3}$

3

14 thirds $= \frac{14}{3}$.

RULE.—*Multiply the whole number by the denominator, add the numerator, and write the denominator under the result.*

Reduce to improper fractions

2. $4\frac{3}{5}$.	Ans. $\frac{23}{5}$.	7. $18\frac{3}{5}$.	Ans. $\frac{93}{5}$.
3. $7\frac{3}{4}$.		8. $21\frac{2}{3}$.	
4. $9\frac{5}{6}$.		9. $19\frac{11}{12}$.	
5. $7\frac{5}{8}$.		10. $25\frac{9}{13}$.	
6. $13\frac{7}{9}$.		11. $35\frac{15}{16}$.	

CASE II.

103. To reduce improper fractions to whole or mixed numbers.

1. How many ones in $\frac{15}{4}$?

SOLUTION.—In *one* there are 4 *fourths*, and in 15 *fourths* there are as many *ones* as 4 is contained times in 15, which are $3\frac{3}{4}$. From this solution we have the following

OPERATION.

$\frac{15}{4} = 15 \div 4 = 3\frac{3}{4}$.

RULE.—*Divide the numerator by the denominator, and the quotient will be the whole or mixed number.*

Reduce to whole or mixed numbers

2. $\frac{9}{4}$.	Ans. $2\frac{1}{4}$	7. $\frac{47}{6}$.	Ans. $7\frac{5}{6}$.
3. $\frac{11}{3}$.		8. $\frac{92}{11}$.	
4. $\frac{19}{5}$.		9. $\frac{75}{12}$.	
5. $\frac{32}{4}$.		10. $\frac{235}{17}$.	
6. $\frac{72}{8}$.		11. $\frac{784}{21}$.	

CASE III.

104. To reduce fractions to higher terms.

1. How many twelfths in $\frac{3}{4}$?

SOLUTION.—Multiplying both numerator and denominator of a fraction by the same number does not change its value, PRIN. V.; hence, multiplying both numerator and denominator by 3, and we have $\frac{3}{4} = \frac{9}{12}$. From this solution we have the following

OPERATION.

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

RULE.—*Multiply both numerator and denominator by any number which will give the required denominator.*

Reduce		Reduce
2. $\frac{2}{3}$ to 12ths.	Ans. $\frac{8}{12}$.	7. $\frac{11}{12}$ to 36ths.
3. $\frac{5}{6}$ to 30ths.	Ans. $\frac{25}{30}$.	8. $\frac{14}{15}$ to 60ths.
4. $\frac{7}{8}$ to 16ths.		9. $\frac{7}{9}$ to 81sts.
5. $\frac{9}{10}$ to 20ths.		10. $\frac{15}{16}$ to 80ths.
6. $\frac{7}{9}$ to 27ths.		11. $\frac{19}{21}$ to 84ths.

CASE IV.

105. To reduce a fraction to lower terms.

106. A fraction is reduced to *lower terms* when it is reduced to one having a smaller numerator and denominator.

107. A fraction is in its *lowest terms* when it cannot be reduced to any lower terms.

1. Reduce $\frac{9}{12}$ to fourths.

SOLUTION.—Dividing both numerator and denominator of a fraction by the same number does not change its value, PRIN. VI.; hence, to reduce $\frac{9}{12}$ to fourths we divide both numerator and denominator by 3, and we have $\frac{3}{4}$. From this solution we have the following

OPERATION.

$$3)\frac{9}{12} = \frac{3}{4}$$

RULE.—1. To reduce a fraction to lower terms, *divide both numerator and denominator by the same number or numbers.*

2. To reduce to lowest terms, *divide in this way until the fraction cannot be reduced any lower.*

Reduce to lowest terms		Reduce to lowest terms	
2. $\frac{6}{9}$, $\frac{14}{21}$.	Ans. $\frac{2}{3}$	7. $\frac{24}{40}$, $\frac{27}{45}$.	Ans. $\frac{3}{5}$.
3. $\frac{8}{12}$, $\frac{12}{18}$.	Ans. $\frac{2}{3}$.	8. $\frac{70}{80}$, $\frac{84}{96}$.	
4. $\frac{10}{12}$, $\frac{25}{30}$.	Ans. $\frac{5}{6}$.	9. $\frac{45}{50}$, $\frac{108}{120}$.	
5. $\frac{16}{24}$, $\frac{18}{27}$.		10. $\frac{99}{108}$, $\frac{121}{132}$.	
6. $\frac{16}{28}$, $\frac{48}{84}$.		11. $\frac{96}{104}$, $\frac{144}{156}$.	

Case V.

108. To reduce compound fractions to simple.

1. What are $\frac{2}{3}$ of $\frac{4}{5}$?

Solution.—$\frac{1}{3}$ of $\frac{4}{5}=\frac{4}{15}$, since multiplying the denominator of a fraction by 3 divides the fraction by 3; and if $\frac{1}{3}$ of $\frac{4}{5}=\frac{4}{15}$, $\frac{2}{3}$ of $\frac{4}{5}$ equals 2 times $\frac{4}{15}$, which are $\frac{8}{15}$. From this solution we have the following

Operation. $\frac{2}{3}\times\frac{4}{5}=\frac{2\times4}{3\times5}=\frac{8}{15}$

Rule.—*Multiply the numerators together, and the denominators together.*

What is		What is	
2. $\frac{3}{4}$ of $\frac{7}{8}$?	Ans. $\frac{21}{32}$.	7. $\frac{11}{12}$ of $\frac{16}{33}$?	
3. $\frac{5}{6}$ of $\frac{7}{9}$?		8. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$?	
4. $\frac{4}{7}$ of $\frac{18}{15}$?		9. $\frac{4}{7}$ of $\frac{9}{10}$ of $\frac{14}{16}$?	
5. $\frac{8}{9}$ of $\frac{11}{12}$?		10. $\frac{3}{5}$ of $\frac{7}{8}$ of $\frac{17}{18}$?	
6. $\frac{5}{8}$ of $\frac{9}{10}$?		11. $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{14}{36}$?	

12. A had $\frac{4}{5}$ of a ton of hay, and sold his neighbor $\frac{2}{3}$ of it; how much did he sell?

Solution.—If A had $\frac{4}{5}$ of a ton of hay, and sold his neighbor $\frac{2}{3}$ of it, he sold his neighbor $\frac{2}{3}$ of $\frac{4}{5}$ of a ton, which is $\frac{8}{15}$ of a ton.

Operation. $\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}$ Ans.

13. A boy picked $\frac{5}{6}$ of a bushel of strawberries, and sold $\frac{2}{3}$ of them; how many did he sell? Ans. $\frac{10}{18}$, or $\frac{5}{9}$.

14. A man had $\frac{5}{6}$ of a bushel of barley, and sold $\frac{3}{4}$ of it; how much did he sell? Ans. $\frac{5}{8}$.

15. A little girl had $\frac{7}{8}$ of a melon, and gave her brother $\frac{4}{7}$ of it; how much did her brother receive? Ans. $\frac{1}{2}$.

16. Says Jennie to Kate, My father owns $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of a ship; what part of the ship did he own? Ans. $\frac{2}{5}$.

COMMON DENOMINATOR.

109. Fractions have a *Common Denominator* when they have the same number for a denominator.

1. Reduce $\frac{3}{4}$ and $\frac{4}{5}$ to a common denominator.

SOLUTION.—Multiplying both numerator and denominator of $\frac{3}{4}$ by 5, the denominator of $\frac{4}{5}$, we have $\frac{15}{20}$; and multiplying both numerator and denominator of $\frac{4}{5}$ by 4, the denominator of $\frac{3}{4}$, we have $\frac{16}{20}$; and this makes the fractions have the same denominator; hence the following

OPERATION.

$$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

$$\frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20}.$$

RULE.—*Multiply both numerator and denominator of each fraction by the denominators of the other fractions.*

For L. C. Denom., *Find the least common multiple of the denominators, divide this by the denominator of each fraction, and multiply both terms of the fraction by the quotient.*

Reduce to a common denominator

2. $\frac{2}{3}$ and $\frac{4}{5}$. Ans. $\frac{10}{15}$, $\frac{12}{15}$.
3. $\frac{4}{5}$ and $\frac{5}{6}$. Ans. $\frac{24}{30}$, $\frac{25}{30}$.
4. $\frac{7}{8}$ and $\frac{4}{5}$.
5. $\frac{3}{4}$ and $\frac{5}{7}$.
6. $\frac{9}{10}$ and $\frac{8}{11}$.
7. $\frac{1}{13}$ and $\frac{1}{14}$.
8. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.
9. $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{4}{5}$.
10. $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{6}{7}$.
11. $\frac{5}{6}$, $\frac{6}{7}$, and $\frac{7}{8}$.

ADDITION OF FRACTIONS.

110. ***Addition of Fractions*** is the process of finding the sum of two or more fractions.

CASE I.

To add when the denominators are alike.

1. What is the sum of $\frac{2}{5}$ and $\frac{4}{5}$?

SOLUTION.—2 *fifths* plus 4 *fifths* equals 6 *fifths*, which equals $1\frac{1}{5}$.

OPERATION.

$$\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$$

2. What is the sum of $\frac{3}{6}$ and $\frac{2}{6}$? Ans. $\frac{5}{6}$.
3. What is the sum of $\frac{5}{6}$ and $\frac{4}{6}$? Ans. $\frac{9}{6}$, or $1\frac{1}{2}$.
4. What is the sum of $\frac{5}{8}$ and $\frac{7}{8}$? Ans. $1\frac{1}{2}$.
5. *What is the* sum of $\frac{7}{9}$ and $\frac{8}{9}$? Ans. $1\frac{5}{9}$, or $1\frac{2}{3}$.

6. Mary had $\frac{2}{5}$ of a dollar and Sarah had $\frac{1}{5}$ of a dollar; how much did they both have?

7. Lucy gave me $\frac{3}{6}$ of a peach, and Fanny gave me $\frac{5}{6}$ of a peach; how much did I receive? Ans. $1\frac{1}{3}$.

8. George and Susie had each $\frac{7}{8}$ of a pine-apple; how much had they together? Ans. $1\frac{3}{4}$.

9. If I walk $\frac{5}{6}$ of a mile and ride $\frac{4}{6}$ of a mile, how far do I go? Ans. $1\frac{1}{2}$ mile.

10. A had $\frac{3}{8}$ of a dollar, B had $\frac{7}{8}$ of a dollar, and C had $\frac{6}{8}$ of a dollar; how much had they all?

CASE II.

To add when the denominators are unlike.

1. What is the sum of $\frac{2}{3}$ and $\frac{3}{4}$?

SOLUTION.—We first reduce the fractions to a common denominator: $\frac{2}{3}=\frac{8}{12}$; $\frac{3}{4}=\frac{9}{12}$; 8 *twelfths* plus 9 *twelfths* are 17 *twelfths*. Hence $\frac{2}{3}+\frac{3}{4}=\frac{17}{12}$. From this we have the following

OPERATION.

$\frac{2}{3}+\frac{3}{4}=$

$\frac{8}{12}+\frac{9}{12}=\frac{17}{12}$

RULE.—*Reduce the fractions to a common denominator; add the numerators, and place the sum over the common denominator.*

NOTE.—Reduce each fraction to its lowest terms before reducing to a common denominator, and also the result after addition.

Find the sum of		Find the sum of	
2. $\frac{2}{3}$ and $\frac{2}{5}$.	Ans. $\frac{16}{15}$.	9. $\frac{7}{8}$ and $\frac{6}{7}$.	Ans. $\frac{97}{56}$.
3. $\frac{3}{4}$ and $\frac{4}{5}$.	Ans. $\frac{31}{20}$.	10. $\frac{3}{9}$ and $\frac{7}{10}$.	Ans. $\frac{31}{30}$.
4. $\frac{3}{4}$ and $\frac{5}{6}$.		11. $\frac{4}{6}$ and $\frac{12}{14}$.	
5. $\frac{4}{5}$ and $\frac{6}{7}$.		12. $\frac{10}{12}$ and $\frac{18}{20}$.	
6. $\frac{3}{7}$ and $\frac{5}{8}$.		13. $\frac{12}{21}$ and $\frac{19}{20}$.	
7. $\frac{2}{9}$ and $\frac{7}{10}$		14. $\frac{16}{17}$ and $\frac{18}{24}$.	
8. $\frac{4}{5}$ and $\frac{8}{9}$.		15. $\frac{19}{20}$ and $\frac{21}{22}$.	

16. A has $\frac{2}{3}$ of a pie, and B $\frac{3}{4}$ of a pie; how much have they both?

17. B having $\frac{3}{4}$ of a ton of hay bought $\frac{4}{5}$ of a ton; how much had he then?

18. Henry owned $\frac{2}{7}$ of a vessel, and bought $\frac{2}{9}$ of the vessel; how much did he then own?

19. A had $7\frac{3}{4}$ dollars, and B gave him $8\frac{2}{3}$ dollars; how many had he then?

NOTE.—Add the 7 and 8, and then add $\frac{3}{4}$ and $\frac{2}{3}$; $8+7=15$; $\frac{3}{4}+\frac{2}{3}=\frac{9}{12}+\frac{8}{12}=\frac{17}{12}=1\frac{5}{12}$; $15+1\frac{5}{12}=16\frac{5}{12}$. Ans.

20. A had $25\frac{2}{5}$ acres of land, and then bought $17\frac{3}{4}$ acres; how many had he then?

21. B had $57\frac{3}{5}$ dollars, and C had $96\frac{5}{8}$ dollars; what was the sum of their money?

22. C sold $96\frac{7}{8}$ yards of cloth, and then had $147\frac{9}{16}$ yards left; how much had he at first?

SUBTRACTION OF FRACTIONS.

111. ***Subtraction of Fractions*** is the process of finding the difference between two fractions.

CASE I.

To subtract when the denominators are alike.

1. Subtract $\frac{3}{8}$ from $\frac{7}{8}$.

SOLUTION.—3 *eighths* subtracted from 7 *eighths* leave 4 *eighths*, and $\frac{4}{8}$ reduced to its lowest terms equals $\frac{1}{2}$.

OPERATION.

$\frac{7}{8}-\frac{3}{8}=\frac{4}{8}$, or $\frac{1}{2}$

2. Subtract $\frac{3}{7}$ from $\frac{5}{7}$.

3. Subtract $\frac{1}{4}$ from $\frac{3}{4}$.

4. Subtract $\frac{2}{9}$ from $\frac{8}{9}$.

5. Subtract $\frac{5}{12}$ from $\frac{11}{12}$.

6. Mary had $\frac{7}{8}$ of an apple, and gave away $\frac{3}{8}$ of an apple; what part of an apple had she left? Ans. $\frac{1}{2}$.

7. Peter found $\frac{4}{5}$ of a dollar, and spent $\frac{2}{5}$ of a dollar; what part of a dollar had he left?

8. If I buy $\frac{11}{12}$ of a ton of hay and sell $\frac{5}{12}$ of a ton, what part of a ton will I have left?

9. Peter has $\frac{2}{10}$ of a dollar, John $\frac{7}{10}$ of a dollar, and

Jacob $\frac{7}{10}$ of a dollar; how much more have Peter and John than Jacob? Ans. $\$\frac{1}{2}$.

10. Mary has $\frac{3}{5}$ of a dollar, Sarah $\frac{4}{5}$ of a dollar, and Jane $\frac{2}{5}$ of a dollar; how much more have Mary and Sarah than Jane? Ans. $1.

Case II.

To subtract when the denominators are unlike.

1. Subtract $\frac{2}{3}$ from $\frac{4}{5}$.

Solution.—We will first reduce the fractions to a common denominator. $\frac{4}{5}=\frac{12}{15}$ and $\frac{2}{3}=\frac{10}{15}$, and 12 *fifteenths* minus 10 *fifteenths* is 2 *fifteenths*. From this solution we have the following

OPERATION.

$\frac{4}{5}-\frac{2}{3}=$
$\frac{12}{15}-\frac{10}{15}=\frac{2}{15}$

Rule.—*Reduce the fractions to a common denominator, subtract the numerators, and place the result over the common denominator.*

Note.—Reduce each fraction, and also the difference, to its lowest terms.

Subtract		Subtract	
2. $\frac{2}{3}$ from $\frac{3}{4}$.	Ans. $\frac{1}{12}$.	8. $\frac{5}{8}$ from $\frac{10}{12}$.	Ans. $\frac{5}{24}$.
3. $\frac{3}{4}$ from $\frac{5}{6}$.	Ans. $\frac{1}{12}$.	9. $\frac{6}{7}$ from $\frac{7}{8}$.	Ans. $\frac{1}{56}$.
4. $\frac{3}{5}$ from $\frac{5}{6}$.		10. $\frac{8}{9}$ from $\frac{13}{14}$.	
5. $\frac{3}{7}$ from $\frac{4}{5}$.		11. $\frac{7}{8}$ from $\frac{11}{12}$.	
6. $\frac{3}{9}$ from $\frac{5}{8}$.		12. $\frac{8}{12}$ from $\frac{9}{10}$.	
7. $\frac{6}{7}$ from $\frac{11}{12}$.		13. $\frac{11}{12}$ from $\frac{14}{15}$.	

14. Mary had $\frac{3}{5}$ of a dollar and gave away $\frac{1}{4}$ of a dollar; how much had she left?

Solution.—If Mary had $\frac{3}{5}$ of a dollar, and gave away $\frac{1}{4}$ of a dollar, she had left the difference between $\frac{3}{5}$ of a dollar and $\frac{1}{4}$ of a dollar, which by reducing to a common denominator and subtracting, we find to be $\frac{7}{20}$ of a dollar

OPERATION.

$\frac{3}{5}-\frac{1}{4}=$
$\frac{12}{20}-\frac{5}{20}=\frac{7}{20}$ Ans.

15. Willie gave Sallie $\frac{4}{5}$ of a quart of peanuts, and Sallie gave him back $\frac{3}{4}$ of a quart; what part of a quart did Sallie keep?

16 A has $\frac{7}{8}$ of a pie; if he gives B $\frac{2}{5}$ of a pie, how much will remain?

17. B bought $\frac{9}{10}$ of a ton of hay, and sold C $\frac{3}{7}$ of a ton; how much did B retain?

18. C owned $\frac{1}{4}$ of a vessel, H bought $\frac{1}{5}$ of this, and then sold $\frac{1}{3}$ of what he bought: how much did H keep?

19 The sum of two fractions is $\frac{19}{20}$, and one fraction is $\frac{1}{10}$: what is the other fraction?

20. If D had $\frac{2}{5}$ of a certain sum of money, and then earned $\frac{3}{4}$ of the same sum, and then spent $\frac{4}{5}$ of the sum, how much remained?

21. The sum of three fractions is $\frac{15}{16}$, and two of these fractions are $\frac{1}{8}$ and $\frac{1}{4}$; what is the third fraction?

22. A has $\frac{7}{8}$ of a sum of money; he owes B $\frac{3}{8}$, and C $\frac{2}{5}$ of that sum of money; how much will A have after paying his debts? Ans. $\frac{1}{10}$.

PRACTICAL PROBLEMS
in Addition and Subtraction of Fractions.

1. Subtract $4\frac{3}{4}$ from $7\frac{1}{4}$.

SOLUTION.—$7\frac{1}{4}$ equals $6+\frac{4}{4}+\frac{1}{4}=6\frac{5}{4}$; $4\frac{3}{4}$ subtracted from $6\frac{5}{4}$ leaves $2\frac{2}{4}$.

OPERATION.

$7\frac{1}{4}=6\frac{5}{4}$

$4\frac{3}{4}$

$2\frac{2}{4}$ Ans.

Subtract	Subtract
2. $5\frac{2}{3}$ from $9\frac{1}{3}$.	5. $16\frac{2}{3}$ from $20\frac{1}{4}$
3. $7\frac{3}{7}$ from $10\frac{2}{7}$.	6. $19\frac{6}{7}$ from $30\frac{2}{3}$.
4. $8\frac{5}{9}$ from $13\frac{2}{9}$.	7. $24\frac{3}{4}$ from $36\frac{3}{5}$.

8. A had $17\frac{3}{5}$ dollars, and gave B $12\frac{4}{5}$; how much remained?

9. A has $6\frac{3}{4}$ dollars, and B has $7\frac{1}{2}$ dollars; how much have both?

10. A read $4\frac{1}{2}$ pages one day and $7\frac{1}{3}$ another day; how many pages did he read in the two days?

11. Mary had 20 dollars; she gave her brother $\$9\frac{3}{4}$ and her sister $\$7\frac{2}{5}$; how much remained?

12. William having \$100 gave \$26$\frac{2}{3}$ to the poor and spent \$18$\frac{5}{8}$ for clothing; how much remained?

13. My father gave me \$3$\frac{3}{5}$, my mother gave me \$5$\frac{3}{4}$, and I then gave my sister \$4$\frac{1}{4}$; how much remained?

14. What is the sum of $\frac{1}{2}$ of $\frac{1}{3}$ and $\frac{1}{3}$ of $\frac{1}{4}$; and what, also, is their difference?

15. I had \$24, and gave $\frac{1}{3}$ of it to my sister and $\frac{1}{4}$ of it to my brother; how much remained?

16. Sarah had \$23, and gave $\frac{1}{4}$ of it to the poor and $\frac{3}{8}$ of it for a dress; how much remained?

17. Henry's father gave him \$16$\frac{3}{4}$, and his mother gave him \$18$\frac{1}{2}$; he then spent $\frac{2}{3}$ of it; how much remained?

18. A man had \$24$\frac{3}{4}$, and then earned \$16$\frac{1}{5}$, and then spent $\frac{1}{2}$ of it; how much remained?

19. Peter had \$17$\frac{1}{2}$, and then lost \$11$\frac{1}{3}$, and then earned \$14; how much had he then?

20. Harold had \$$\frac{4}{5}$, then lost \$$\frac{3}{4}$, and then earned \$$\frac{5}{6}$; how much had he then?

MULTIPLICATION OF FRACTIONS.

112. ***Multiplication of Fractions*** is the process of multiplying when one or both terms are fractions.

CASE I.

113. To multiply a fraction by a whole number.

1. Multiply $\frac{5}{8}$ by 4.

SOLUTION.—4 times $\frac{5}{8}$ equal $\frac{20}{8}$, according to PRIN. I. Or, 4 times $\frac{5}{8}$ equal $\frac{5}{2}$, since dividing the denominator multiplies the fraction, according to PRIN. IV. From this we have the following

OPERATION.

$\frac{5}{8} \times 4 = \frac{20}{8}$

or, $\frac{5}{8} \times 4 = \frac{5}{2}$

RULE.—*To multiply a fraction by an integer, multiply the numerator or divide the denominator by the integer.*

Multiply		Multiply	
2. $\frac{9}{10}$ by 5.	Ans. $\frac{9}{2}$.	6. $\frac{17}{18}$ by 9.	Ans. $8\frac{1}{2}$.
3. $\frac{11}{16}$ by 4.	Ans. $\frac{11}{4}$.	7. $\frac{20}{21}$ by 7.	Ans. $6\frac{2}{3}$.
4. $\frac{14}{15}$ by 7.		8. $\frac{25}{72}$ by 12.	
5. $\frac{18}{19}$ by 3.		9. $\frac{121}{144}$ by 36.	

10. A has $\frac{17}{18}$ of a ton of hay, and B has 3 times as much; how much have both?

Case II.

114. To multiply a whole number by a fraction.

1. Multiply 8 by $\frac{2}{3}$; also by $4\frac{2}{3}$.

Solution 1.—8 multiplied by $\frac{1}{3}$ equals $\frac{1}{3}$ of 8, or $\frac{8}{3}$, and 8 multiplied by $\frac{2}{3}$ equals 2 times $\frac{8}{3}$, or $\frac{16}{3}$.

OPERATION.

$$8 \times \tfrac{2}{3} = \frac{8 \times 2}{3}$$
$$= \tfrac{16}{3} = 5\tfrac{1}{3}$$

Solution 2.—We multiply 8 by 2 and divide by 3, and have $5\frac{1}{3}$; then multiply by 4 and add the product 32 to $5\frac{1}{3}$, making $37\frac{1}{3}$. Hence, in a mixed number we multiply first by the fraction, and then by the integer. From these solutions we have the following

OPERATION.

$$\begin{array}{r} 8 \\ 4\frac{2}{3} \\ \hline 3)16 \\ \hline 5\frac{1}{3} \\ 32 \\ \hline 37\frac{1}{3} \end{array}$$

Rule.—*Multiply the whole number by the numerator of the fraction, and divide the product by the denominator.*

Multiply		Multiply	
2. 16 by $\frac{3}{4}$.	Ans. 12.	7. 45 by $\frac{8}{9}$.	Ans. 40.
3. 18 by $\frac{5}{6}$.	Ans. 15.	8. 43 by $\frac{5}{8}$.	Ans. $35\frac{5}{8}$
4. 12 by $\frac{7}{8}$.		9. 28 by $5\frac{4}{7}$.	
5. 20 by $2\frac{9}{10}$.		10. 76 by $4\frac{7}{8}$.	
6. 35 by $4\frac{6}{7}$.		11. 85 by $8\frac{9}{10}$.	

12. A has 18 tons of hay, and B has $4\frac{3}{4}$ times as much plus $3\frac{1}{2}$ tons; how much has B?

Case III.

115. To multiply a fraction by a fraction.

1. *Multiply* $\frac{3}{5}$ by $\frac{3}{4}$.

SOLUTION.—$\frac{2}{5}$ multiplied by $\frac{1}{4}$ equals $\frac{1}{4}$ of $\frac{2}{5}$, which is $\frac{2}{20}$, according to PRIN. III.; and $\frac{3}{5}$ multiplied by $\frac{1}{4}$ equals 3 times $\frac{2}{20}$ or $\frac{6}{20}$.

OPERATION.

$$\frac{2}{5} \times \frac{3}{4} = \frac{3 \times 3}{5 \times 4} = \frac{6}{20}.$$

RULE.—*Multiply the numerators together for the numerator, and the denominators together for the denominator of the product.*

NOTE.—Reduce the result to its lowest terms.

What is the product of

2. $\frac{3}{4}$ by $\frac{2}{5}$? Ans. $\frac{3}{10}$.
3. $\frac{7}{8}$ by $\frac{4}{5}$? Ans. $\frac{7}{10}$.
4. $\frac{9}{10}$ by $\frac{5}{6}$?
5. $\frac{11}{12}$ by $\frac{9}{14}$?
6. $\frac{18}{20}$ by $\frac{15}{18}$?
7. $\frac{19}{21}$ by $\frac{7}{8}$? Ans $\frac{19}{24}$.
8. $\frac{20}{21}$ by $\frac{28}{25}$? Ans. $\frac{16}{15}$.
9. $\frac{25}{29}$ by $\frac{17}{35}$? Ans. $\frac{85}{203}$.
10. $\frac{2}{3}$ by $\frac{3}{4}$ of $\frac{5}{6}$?
11. $\frac{7}{8}$ of $\frac{4}{5}$ by $\frac{25}{28}$?
12. A has $\frac{7}{8}$ of a ton of hay, and B has $\frac{6}{7}$ as much plus $2\frac{3}{5}$ tons; how much has B?

DIVISION OF FRACTIONS.

116. ***Division of Fractions*** is the process of dividing when one or both terms are fractional.

CASE I.

117. To divide when the dividend is a fraction.

1. Divide $\frac{8}{9}$ by 4.

SOLUTION.—$\frac{8}{9}$ divided by 4 equals $\frac{2}{9}$, according to PRIN. I. When the numerator will not contain the divisor, we multiply the denominator, according to PRIN. III.

OPERATION.

$$\frac{8}{9} \div 4 = \frac{2}{9}$$

RULE.—*Divide the numerator, or multiply the denominator, by the divisor.*

Divide

2. $\frac{9}{10}$ by 3. Ans. $\frac{3}{10}$.
3. $\frac{8}{11}$ by 4. Ans. $\frac{2}{11}$.
4. $\frac{12}{13}$ by 6.
5. $\frac{9}{11}$ by 4.
6. $\frac{7}{8}$ by 8.
7. $\frac{12}{13}$ by 7. Ans. $\frac{12}{91}$.
8. $\frac{16}{17}$ by 5 Ans. $\frac{16}{85}$.
9. $\frac{18}{7}$ by 8.
10. $3\frac{1}{4}$ by 9.
11. $5\frac{2}{5}$ by 12.

12. A gave $3\frac{1}{5}$ dollars to 6 little girls; how much did each receive?

CASE II.

118. To divide when the divisor is a fraction.

1. Divide $\frac{3}{4}$ by $\frac{4}{5}$.

SOLUTION.—$\frac{3}{4}$ divided by 1 equals $\frac{3}{4}$, hence $\frac{3}{4}$ divided by $\frac{1}{5}$ equals 5 times $\frac{3}{4}$, and $\frac{3}{4}$ divided by $\frac{4}{5}$ equal $\frac{1}{4}$ of 5 times $\frac{3}{4}$, or $\frac{5}{4}$ times $\frac{3}{4}$, which equal $\frac{15}{16}$. Hence, we see the divisor becomes inverted, and we have the following

OPERATION.

$\frac{3}{4} \div \frac{4}{5} =$

$\frac{3}{4} \times \frac{5}{4} = \frac{15}{16}$

RULE.—*Invert the divisor, and multiply the dividend by the resulting fraction.*

Divide		Divide	
2. $\frac{6}{7}$ by $\frac{3}{4}$.	Ans. $\frac{8}{7}$.	8. $\frac{15}{16}$ by $\frac{9}{12}$.	Ans. $1\frac{1}{4}$.
3. $\frac{4}{5}$ by $\frac{2}{3}$.	Ans. $\frac{6}{5}$.	9. $\frac{21}{32}$ by $\frac{14}{18}$.	Ans. $\frac{27}{32}$.
4. $\frac{7}{8}$ by $\frac{5}{6}$.	Ans. $\frac{21}{20}$.	10. $\frac{20}{21}$ by $\frac{18}{35}$.	Ans. $1\frac{2}{7}\frac{3}{7}$.
5. $\frac{9}{10}$ by $\frac{3}{8}$.		11. $\frac{16}{17}$ by $\frac{8}{21}$.	Ans. $2\frac{8}{17}$.
6. $\frac{11}{12}$ by $\frac{8}{9}$.		12. $\frac{12}{15}$ by $\frac{16}{35}$.	
7. $\frac{10}{11}$ by $\frac{11}{10}$.		13. $\frac{32}{36}$ by $\frac{48}{50}$.	

14. How much cloth will $\$4\frac{1}{2}$ buy, at $\$\frac{3}{4}$ per yard?

REDUCTION OF COMPLEX FRACTIONS.

118½. A ***Complex Fraction*** is one whose numerator or denominator or both are fractional.

1. Reduce $\frac{\frac{2}{3}}{\frac{4}{5}}$ to a simple fraction.

SOLUTION.—This fraction means that $\frac{2}{3}$ is to be divided by $\frac{4}{5}$, and inverting the divisor and multiplying we have $\frac{2}{3} \times \frac{5}{4}$ which equals $\frac{5}{6}$.

OPERATION.

$\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{5}{6}$

RULE.—*Multiply the numerator of the complex fraction by its denominator inverted.*

Or, *Reduce mixed numbers, if any, to fractions; then multiply the numerator of the upper fraction by the denominator of the lower fraction, and the denominator of the upper fraction by the numerator of the lower.*

2. Reduce $\frac{\frac{3}{4}}{\frac{5}{6}}$. Ans. $\frac{9}{10}$.

3. Reduce $\frac{4}{\frac{2}{3}}$. Ans. 6.

4. Reduce $\frac{4}{1\frac{1}{3}}$. Ans. 3.

5. Reduce $\frac{3\frac{1}{3}}{2\frac{1}{2}}$. Ans. $1\frac{1}{3}$.

6. Reduce $\frac{\frac{2}{3}\text{ of }\frac{3}{4}}{3\frac{1}{3}}$. Ans. $\frac{3}{20}$.

7. Reduce $\frac{\frac{2}{3}\text{ of }\frac{4}{5}}{\frac{4}{5}\text{ of }\frac{5}{6}}$. Ans. $\frac{4}{5}$.

8. Reduce $\frac{5\frac{1}{4}}{2+1\frac{1}{2}}$. A. $1\frac{1}{2}$

9. Reduce $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{3}-\frac{1}{4}}$. Ans. 10.

MISCELLANEOUS EXAMPLES.

Reduce to improper fractions.

1. $6\frac{7}{8}$. Ans. $\frac{55}{8}$.
2. $9\frac{5}{7}$. Ans. $\frac{68}{7}$.
3. $12\frac{7}{9}$. Ans. $\frac{115}{9}$.
4. $13\frac{13}{14}$. Ans. $\frac{195}{14}$.
5. $24\frac{24}{25}$. Ans. $\frac{624}{25}$.
6. $35\frac{17}{69}$. Ans. $\frac{2432}{69}$.
7. $132\frac{53}{85}$. Ans. $\frac{11273}{85}$.
8. $345\frac{69}{113}$. Ans. $\frac{39054}{113}$.
9. $547\frac{147}{1268}$. Ans. $\frac{693743}{1268}$.
10. $777\frac{654}{987}$. Ans. $\frac{767553}{987}$.

Reduce to mixed numbers

11. $\frac{146}{12}$. Ans. $12\frac{1}{6}$.
12. $\frac{178}{13}$. Ans. $13\frac{9}{13}$.
13. $\frac{192}{14}$. Ans. $13\frac{5}{7}$.
14. $\frac{157}{48}$. Ans. $3\frac{13}{48}$.
15. $\frac{408}{124}$. Ans. $3\frac{9}{31}$.
16. $\frac{607}{225}$. Ans. $2\frac{157}{225}$.
17. $\frac{1728}{345}$. Ans. $5\frac{1}{115}$.
18. $\frac{20081}{483}$. Ans. $41\frac{278}{483}$.
19. $\frac{8305740}{5726}$. Ans. $145\frac{1852}{2868}$.
20. $\frac{800009}{60501}$. Ans. $13\frac{19288}{8643}$.

Reduce to lowest terms

21. $\frac{121}{132}$. Ans. $\frac{11}{12}$.
22. $\frac{132}{144}$. Ans. $\frac{11}{12}$.
23. $\frac{144}{180}$. Ans. $\frac{4}{5}$.
24. $\frac{182}{196}$. Ans. $\frac{13}{14}$.
25. $\frac{3136}{405}$. Ans. $\frac{7}{9}$.
26. $\frac{543}{840}$. Ans. $\frac{181}{280}$.
27. $\frac{480}{1728}$. Ans. $\frac{5}{18}$.
28. $\frac{24240}{48480}$. Ans. $\frac{1}{2}$.
29. $\frac{1320}{16880}$. Ans. $\frac{11}{14}$.
30. $\frac{9072}{12096}$. Ans. $\frac{3}{4}$.

Reduce to simple fractions

31. $\frac{3}{4}$ of $\frac{6}{7}$. Ans. $\frac{9}{14}$.
32. $\frac{5}{8}$ of $\frac{12}{13}$. Ans. $\frac{15}{26}$.
33. $\frac{8}{9}$ of $\frac{15}{16}$. Ans. $\frac{5}{6}$.
34. $\frac{2}{5}$ of $4\frac{1}{6}$. Ans. $1\frac{2}{3}$.
35. $\frac{3}{7}$ of $8\frac{2}{5}$. Ans. $3\frac{3}{5}$.
36. $\frac{5}{8}$ of $8\frac{4}{5}$. Ans. $5\frac{1}{2}$.
37. $\frac{13}{14}$ of $\frac{16}{39}$. Ans. $\frac{8}{21}$.
38. $\frac{15}{16}$ of $\frac{24}{25}$. Ans. $\frac{9}{10}$.
39. $7\frac{17}{18}$ of $\frac{27}{35}$. Ans. $6\frac{9}{70}$.
40. $125\frac{53}{81}$ of $44\frac{43}{56}$. Ans. $611\frac{1}{4}$

Find the value of

41. $\frac{4}{5} + \frac{5}{6}$. Ans. $1\frac{19}{30}$.
42. $\frac{5}{6} + \frac{7}{8}$. Ans. $1\frac{17}{24}$.
43. $\frac{3}{10} + \frac{11}{12}$. Ans. $1\frac{13}{60}$.
44. $4\frac{3}{4} + 5\frac{4}{5}$. Ans. $10\frac{11}{20}$.
45. $6\frac{5}{6} + 7\frac{7}{8}$. Ans. $14\frac{17}{24}$.
46. $9\frac{3}{4} + 8\frac{7}{10}$. Ans. $18\frac{9}{20}$.
47. $\frac{2}{3} + \frac{3}{4} + \frac{5}{8}$. Ans. $2\frac{1}{24}$.
48. $\frac{4}{7} + \frac{9}{10} + \frac{11}{14}$. A. $2\frac{29}{35}$.
49. $\frac{11}{12} + \frac{14}{15} + \frac{15}{16}$. A. $2\frac{63}{80}$.
50. $\frac{14}{15} + \frac{23}{40} + \frac{77}{88}$. A. $2\frac{23}{60}$.

Find the value of

51. $\frac{8}{9} - \frac{5}{6}$. Ans. $\frac{1}{18}$.
52. $\frac{11}{12} - \frac{3}{8}$. Ans. $\frac{13}{24}$.
53. $\frac{13}{14} - \frac{13}{21}$. Ans. $\frac{13}{42}$.
54. $8\frac{2}{3} - 5\frac{3}{4}$. Ans. $2\frac{11}{12}$.
55. $9\frac{3}{5} - 6\frac{3}{8}$. Ans. $3\frac{9}{40}$.
56. $12\frac{7}{9} - 10\frac{5}{6}$. Ans. $1\frac{17}{18}$.
57. $\frac{3}{8} + \frac{5}{9} - \frac{3}{4}$. Ans. $\frac{13}{72}$.
58. $\frac{6}{7} + \frac{5}{8} - \frac{7}{12}$. Ans. $\frac{151}{168}$.
59. $\frac{8}{9} - \frac{7}{8} + \frac{9}{10}$. Ans. $\frac{329}{360}$.
60. $\frac{10}{11} - \frac{5}{13} - \frac{67}{143}$. A. $\frac{8}{143}$.

Find the value of

61. $\frac{3}{4} \times 6$. Ans. $4\frac{1}{2}$.
62. $\frac{5}{6} \times 8$. Ans. $6\frac{2}{3}$.
63. $\frac{7}{8} \times 12$. Ans. $10\frac{1}{2}$.
64. $5\frac{3}{4} \times 8$. Ans. 46.
65. $7\frac{8}{9} \times 12$. Ans. $94\frac{2}{3}$.
66. $8\frac{5}{9} \times 15$. Ans. $128\frac{1}{3}$.
67. $\frac{5}{7} \times \frac{6}{8} \times \frac{14}{15}$. Ans. $\frac{1}{2}$.
68. $\frac{11}{12} \times \frac{15}{16} \times \frac{18}{22}$. A. $\frac{45}{64}$.
69. $\frac{16}{21} \times \frac{18}{25} \times \frac{35}{36}$. A. $\frac{8}{15}$.
70. $\frac{27}{28} \times \frac{49}{54} \times \frac{64}{91}$. A. $\frac{8}{13}$.

Find the value of

71. $\frac{6}{7} \div 4$. Ans. $\frac{3}{14}$.
72. $\frac{8}{9} \div 6$. Ans. $\frac{4}{27}$.
73. $\frac{12}{13} \div 8$. Ans. $\frac{3}{26}$.
74. $4 \div \frac{6}{7}$. Ans. $4\frac{2}{3}$.
75. $16 \div \frac{8}{9}$. Ans. 18.
76. $5\frac{1}{3} \div 12$. Ans. $\frac{4}{9}$.
77. $\frac{11}{12} \div 5\frac{1}{2}$. Ans. $\frac{1}{6}$.
78. $\frac{14}{15} \div 3\frac{1}{2}$. Ans. $\frac{4}{15}$.
79. $6\frac{7}{8} \div 8\frac{1}{4}$. Ans. $\frac{5}{6}$.
80. $9\frac{5}{7} \div 12\frac{3}{14}$. Ans. $\frac{136}{171}$.

Find the value of

81. $\frac{\frac{3}{4}}{3\frac{3}{4}}$. Ans. $\frac{1}{5}$.
82. $\frac{\frac{4}{5}}{4\frac{4}{5}}$. Ans. $\frac{1}{6}$.
83. $\frac{\frac{5}{6}}{6\frac{1}{4}}$. Ans. $\frac{2}{15}$.
84. $\frac{8\frac{3}{4}}{9\frac{1}{6}}$. Ans. $\frac{21}{22}$.
85. $\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{3} + \frac{1}{4}}$. Ans. $1\frac{3}{7}$.
86. $\frac{\frac{1}{3} - \frac{1}{5}}{\frac{1}{4} - \frac{1}{6}}$. Ans. $1\frac{3}{5}$.
87. $\frac{\frac{2}{3}}{\frac{4}{5}} + \frac{1\frac{1}{2}}{2\frac{1}{4}}$. Ans. $1\frac{1}{2}$
88. $\frac{5\frac{1}{3}}{2\frac{2}{5}} + \frac{6\frac{1}{4}}{4\frac{3}{8}}$. Ans. $3\frac{41}{68}$.
89. $\frac{4\frac{3}{4}}{7\frac{2}{5}} \div \frac{6\frac{1}{3}}{5\frac{2}{7}}$. Ans. $\frac{15}{28}$.
90. $\frac{6\frac{2}{3}}{9\frac{5}{6}} \times \frac{7\frac{3}{4}}{8\frac{1}{5}}$. Ans. $\frac{775}{1368}$.

PRACTICAL PROBLEMS.

1. What cost 24 apples at $\frac{3}{4}$ of a cent each?
Ans. 18 cents.

2. What cost 45 oranges at $2\frac{2}{5}$ cents a piece?
Ans. $1.08.

3. What cost $16\frac{2}{3}$ yards of muslin at $12\frac{1}{2}$ cents a yard?
Ans. 2.08\frac{1}{3}$.

4. What cost 6 quarts of berries at $18\frac{3}{4}$ cents a quart?
Ans. 1.12\frac{1}{2}$.

5. What cost 8 yards of ribbon at $12\frac{1}{2}$ cents a yard?
Ans. $1.00.

6. What cost 4 bushels of oats at $62\frac{1}{2}$ cents a bushel?
Ans. $2.50.

7. What cost 3 quarts of nuts at $8\frac{1}{3}$ cents a quart?
Ans. 25 cents.

8. What cost 12 bushels of apples at $\$\frac{3}{4}$ a bushel?
Ans. $9.

9. What cost $6\frac{3}{4}$ yards of cloth at $\$2\frac{2}{3}$ a yard?
Ans. $18.

10. What cost $13\frac{1}{2}$ pounds of fish at $9\frac{3}{4}$ cents a pound?
Ans. 1.31\frac{5}{8}$.

11. What cost $18\frac{3}{4}$ yards of ribbon at $31\frac{1}{4}$ cents a yard?
Ans. 5.85\frac{15}{16}$.

12. What cost $26\frac{3}{8}$ pounds of raisins, at $18\frac{3}{4}$ cents a pound? Ans. 4.94\frac{17}{32}$.

13. If one yard of cloth cost $8, how many yards can be bought for $47? Ans. $5\frac{7}{8}$ yards.

14. A boy had $\$5\frac{3}{4}$ and found $\$6\frac{2}{5}$; how much money had he then? Ans. $12.15.

15. A man had $9\frac{1}{2}$ tons of hay and sold $4\frac{3}{4}$ tons of it; how much had he left? Ans. $4\frac{3}{4}$ tons.

16. A had $27\frac{2}{3}$ acres of land and bought $21\frac{1}{4}$ acres; how much land had he then? Ans. 48\frac{11}{12}$ acres.

17. *The sum of two fractions is* $\frac{17}{18}$ *and one fraction is* $\frac{1}{2}$*; what is the other fraction?* Ans. $\frac{1}{9}$.

18. A man earned $25\frac{3}{4}$ and spent $18\frac{1}{5}$; how much money remained? Ans. $7.55.

19. Peter had $26\frac{1}{2}$ and gave $12\frac{3}{4}$ to his sister; how much did he keep? Ans. $13.75.

20. A. sold 5 bushels more than $\frac{1}{3}$ of 40 bushels of apples; how many bushels remained? Ans. $21\frac{2}{3}$.

21. Mary had $25 and spent $\frac{1}{2}$ of it for a dress, and $\frac{1}{2}$ of the remainder for a bonnet; how much then remained? Ans. $6.25.

22. A boy earned $18\frac{3}{4}$ and then had $45\frac{1}{2}$; how much had he at first? Ans. $26.75.

23. How many bushels of potatoes can be bought for $15 at $\$\frac{3}{4}$ a bushel? Ans. 20 bushels.

24. How many pounds of tea, at $1\frac{1}{5}$ a pound can be bought for $40\frac{1}{2}$? Ans. 36 lb.

25. A man bought $12\frac{1}{2}$ yards of cloth for $62\frac{1}{2}$; what did he pay a yard? Ans. $5.

26. How many tons of coal, at $6\frac{3}{4}$ a ton, can be bought for $72? Ans. $10\frac{2}{3}$ tons.

27. If $8\frac{3}{4}$ pounds of grapes cost 49 cents, how much is that a pound? Ans. $5\frac{3}{5}$ cents.

28. How many yards of cloth, at $5\frac{5}{8}$ a yard, can be bought for $18\frac{3}{4}$? Ans. $3\frac{1}{3}$ yards.

29. How many yards of tape, at $6\frac{1}{4}$ cents a yard, can be bought for $58\frac{3}{4}$ cents? Ans. $9\frac{2}{5}$ yards.

30. How many bushels of wheat, at $1\frac{3}{5}$ dollars a bushel, can be bought for $242\frac{2}{5}$? Ans. $151\frac{1}{2}$ bushels.

31. How many sheep, at $8\frac{3}{4}$ a head, can be bought for $157\frac{1}{2}$? Ans. 18.

32. A lady bought $25\frac{1}{2}$ yards of muslin for $6.24\frac{3}{4}$; what was the price per yard? Ans. $24\frac{1}{2}$ cents.

33. How much land can be bought for $543\frac{1}{4}$, at $43\frac{1}{2}$ an acre? Ans. $12\frac{85}{174}$ acres.

34. A servant girl bought $15\frac{1}{8}$ pounds of meat for $2.18\frac{3}{4}$; what was the price a pound? Ans. $14\frac{56}{171}$ cents.

35. A man paid \$1566 for cows, giving \$65¼ a head; how many did he buy? Ans. 24.

36. How many yards of muslin, at 16⅔ cents a yard, can you buy for \$2.08⅓? Ans. 12½ yards.

37. The product of two fractions is $\frac{8}{9}$, and one fraction is $\frac{14}{15}$; what is the other fraction? Ans. $\frac{45}{112}$.

38. What will 3571 feet of lumber cost at \$30⅝ per thousand? Ans. \$109.36$\frac{3}{16}$.

39. The quotient of two numbers is $\frac{19}{20}$, and the divisor is $\frac{9\frac{1}{2}}{20}$; what is the dividend? Ans. $\frac{361}{800}$.

40. If the receipts of the Pennsylvania Railroad for one year are \$3,542,000, and the expenses are $\frac{60\frac{37}{100}}{100}$ of the receipts, what are the expenses? A. \$2,138,305.40.

ARITHMETICAL ANALYSIS.

119. ***Analysis*** is the process of solving problems by a comparison of their elements. In comparing, we reason to the unit and from the unit, the unit being the basis of the reasoning process.

Case I.

120. To pass from one integer to another.

1. If 5 cows cost \$80, what will 7 cows cost at the same rate?

Solution.—If 5 cows cost \$80, *one* cow costs ⅕ of \$80, which is \$16, and 7 cows will cost 7 times \$16, which are \$112.

OPERATION.

5)80
16
7
112 Ans.

2. If 6 hens cost 186 cents, what will 9 hens cost at the same rate?

3. If 5 pigs cost \$35, what will 11 pigs cost at the same rate?

4. If 8 horses cost $1200, what will 12 horses cost at the same rate?

5. If 7 yards of cloth cost $42, what will 25 yards cost at the same rate?

6. How much must I pay for 36 cows, at the rate of 7 cows for 196 dollars?

7. What will 17 books cost, at the rate of 8 books for $10.80?

8. A man bought 72 ducks at the rate of $21 for 7; what did they cost?

9. If a man can walk 324 miles in 9 days, how far can he walk in 69 days?

10. In 26 years there are 9490 days; how many days are there in 75 years?

11. In 5 square miles there are 3200 acres; how many acres in 64 square miles?

12. If a car run 2736 miles in 18 days, how far will it run in 54 days?

Case II.

121. To pass from a fraction to an integer.

1. If $\frac{2}{3}$ of an acre of land cost $96, what will one acre cost?

Solution.—If $\frac{2}{3}$ of an acre cost $96, $\frac{1}{3}$ of an acre cost $\frac{1}{2}$ of $96, or $48, and if $\frac{1}{3}$ of an acre cost $48, $\frac{3}{3}$ of an acre, or one acre, will cost 3 times $48, or $144.

Operation

$\frac{2}{3} = \$96$

$\frac{1}{3} = \$48$

$\frac{3}{3} = \$144$ Ans.

2. If $\frac{3}{4}$ of a sum of money is $72, required the sum.

3. If $\frac{5}{6}$ of the cost of a cow is $25, required the cost of the cow.

4. What cost 2 boxes of raisins, if $\frac{3}{5}$ of a box cost 6 dollars?

5. What is the distance from Lancaster to Philadelphia, if $\frac{3}{4}$ of the distance is 51 miles?

6. If the cost of $\frac{5}{8}$ of an acre of land is $120, what will 4 acres cost at the same rate?

7. If $\frac{6}{7}$ of a farm cost $7200, what will the whole farm cost at that rate?

8. How much will 7 loads of hay weigh, if $\frac{7}{9}$ of a load weighs 840 pounds?

9. What will 17 horses cost me, if $\frac{3}{7}$ of the price of a horse is 93 dollars?

10. A merchant bought 236 barrels of flour at the rate of $8 for $\frac{4}{5}$ of a barrel; how much did they cost him?

Case III.

122. To pass from a unit or fraction to a fraction.

1. If one barrel of flour costs $12, what will $\frac{3}{4}$ of a barrel cost?

Solution.—If one barrel of flour costs $12, 1 fourth of a barrel will cost $\frac{1}{4}$ of $12, or $3, and $\frac{3}{4}$ of a barrel will cost 3 times $3, or $9.

Operation.

4)12
3
3
9 Ans.

2. If one acre of land is worth $125, what is $\frac{4}{5}$ of an acre worth?

3. A paid $1650 for a pleasure-boat; how much would he have paid if he had given $\frac{5}{6}$ as much?

4. If $\frac{2}{3}$ of a barrel of flour cost $8, what will $\frac{3}{4}$ of a barrel cost?

5. If there are 40 pounds in $\frac{2}{3}$ of a bushel of clover-seed, how many pounds are there in $\frac{5}{8}$ of a bushel?

6. If there are 50 pounds in $\frac{5}{8}$ of a bushel of wheat, how many pounds are there in $\frac{1}{1}\frac{1}{2}$ of a bushel?

7. If there are 49 pounds in $\frac{7}{8}$ of a bushel of rye, how many pounds are there in $\frac{5}{7}$ of a bushel?

8. If there are 147 pounds in $\frac{3}{4}$ of a barrel of flour, how many pounds are there in $\frac{6}{7}$ of a barrel?

9. If there are 154 cubic inches in $\frac{2}{5}$ of a gallon, how *many cubic* inches in $\frac{5}{7}$ of a gallon?

10. If there are 1536 cubic inches in $\frac{8}{9}$ of a cubic foot, how many cubic inches in $1\frac{1}{2}$ of a cubic foot?

Case IV.

123. Given a fractional part and the remainder, to find the whole.

1. A man spent $\frac{3}{5}$ of his money, and then had \$24 remaining; how much money had he at first?

Solution.—If he spent $\frac{3}{5}$ of his money, there remained $\frac{5}{5}$ of his money minus $\frac{3}{5}$ of his money, which is $\frac{2}{5}$ of his money, which is \$24. If $\frac{2}{5}$ of his money is \$24, $\frac{1}{5}$ of his money is $\frac{1}{2}$ of \$24, which is \$12. and $\frac{5}{5}$ of his money is 5 times \$12, or \$60.

OPERATION.

$\frac{5}{5} - \frac{3}{5} = \frac{2}{5} = \$24$

$\frac{1}{5} = \$12$

$\frac{5}{5} = \$60$ Ans.

2. A man spent $\frac{3}{5}$ of his money, and then had \$30 remaining; how much had he at first?

3. William sold $\frac{2}{5}$ of his hens, and then had 60 remaining; how many had he at first?

4. Henry sold $\frac{2}{7}$ of his bank-stock, and the remainder was worth \$550; how much had he at first?

5. After giving $\frac{1}{4}$ of his income to the poor, Samuel had \$960 remaining; what was his income?

6. A pole stands $\frac{1}{3}$ in the mud and $\frac{1}{4}$ in the water, and 12 feet in the air; required the length of the pole.

7. One-fourth of a drove of animals are cows, $\frac{1}{5}$ are pigs, and the remainder, 132, are sheep; how many animals in the drove?

8. Two-fifths of my money is in bank, $\frac{1}{3}$ in government bonds, and \$480 in cash; what was my money?

9. A sold $\frac{1}{4}$ of his land to B, and $\frac{3}{7}$ to C, and then had 90 acres remaining; how much had he at first?

10. A man walked $\frac{2}{7}$ of the distance from Lancaster to Philadelphia one day, $\frac{2}{5}$ of the distance the next day, and the remaining distance, 22 miles, the third day; how far did he walk each day? Ans. 20; 28; 22.

SECTION VI.

DECIMAL FRACTIONS.

124. A ***Decimal Fraction*** is a number of the decimal divisions of a unit; that is, a number of *tenths, hundredths*, etc.

125. A decimal fraction is usually expressed by placing a point before the numerator and omitting the denominator. Thus, .5 represents $\frac{5}{10}$; .05 represents $\frac{5}{100}$, etc.

126. The point is called the *decimal point*, or *separatrix*. The decimal fraction thus expressed is called a *decimal*.

127. This method of expressing decimal fractions is but an extension of the method of notation for integers. This method, as applied to integers and fractions, is exhibited in the following

NOTATION AND NUMERATION TABLE.

Places.	8th,	7th,	6th,	5th,	4th,	3d,	2d,	1st,	2d,	3d,	4th,	5th,	6th,	7th,	8th,
	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.
	6	6	6	6	6	6	6	6.	6	6	6	6	6	6	6

EXAMPLES IN NUMERATION.

1. Read the decimal .36.

SOLUTION.—This expresses 3 tenths and 6 hundredths, or, since *3 tenths* equals 30 hundredths, and 30 hundredths plus 6 hundredths *equal 36* hundredths, it may also be read 36 hundredths.

128. Hence there are two methods of reading decimals, which are expressed by the following rules:—

RULE 1.—*Commencing at tenths, read each figure in order toward the right, giving it its proper denomination.*

RULE 2.—*Read the decimal as a whole number, and give it the denomination of the last figure on the right; numerating toward the point to determine the numerator. and from the point to determine the denominator.*

Read the following decimals:—

2. .45.	6. .046.	10. 2.0123.
3. .83.	7. .007.	11. 4.2057.
4. .126.	8. .3216.	12. 13.0205.
5. .324.	9. .1357.	13. 27.0027.

EXAMPLES IN NOTATION.

1. Express 25 hundredths in the form of a decimal.

SOLUTION.—25 hundredths equals 2 tenths and 5 hundredths, and this is expressed by writing a decimal point before 25, thus, .25 Hence the following

RULE.—*Write the decimal as we would a whole number, placing the decimal point so as to give each figure its proper place, using ciphers after the decimal point if necessary.*

Express the following in the decimal form.

2. Thirty-four hundredths.
3. Seventy-five hundredths.
4. Two-tenths and six-hundredths.
5. Twenty-five thousandths.
6. Four-tenths and 7-thousandths.
7. Seven-tenths and 8-thousandths.
8. Five hundred and 25-thousandths.
9. Three-tenths and 7 ten-thousandths.
10. Four hundredths and 96 millionths. Ans. .040096.

PRINCIPLES OF DECIMAL NOTATION.

129. We now present the following principles of *Decimals, which* the pupils will illustrate.

1. *Changing the decimal point one place toward the right multiplies by* 10; *two places, by* 100, etc.

2. *Changing the decimal point one place toward the left divides by* 10; *two places, by* 100, etc.

3. *Placing a cipher between the decimal point and a decimal divides the decimal by ten.*

REDUCTION OF DECIMALS.

130. The ***Reduction of Decimals*** is the process of changing their form without changing their value.

There are two cases:—

1. To reduce decimals to common fractions.
2. To reduce common fractions to decimals.

CASE I.

131. To reduce a decimal to a common fraction.

1. Reduce .45 to a common fraction.

SOLUTION.—.45 expressed in the form of a common fraction, is $\frac{45}{100}$, which, reduced to its lowest terms, equals $\frac{9}{20}$. Hence we have the following

OPERATION.

$.45 = \frac{45}{100}$

$\quad = \frac{9}{20}$ Ans.

RULE.—*Write the denominator under the decimal, omitting the decimal point, and reduce the common fraction to its lowest terms.*

Reduce the following decimals to common fractions.

2. .35.	Ans. $\frac{7}{20}$.	6. 9.75.	Ans. $9\frac{3}{4}$.
3. .48.	Ans. $\frac{12}{25}$.	7. .725.	Ans. $\frac{29}{40}$.
4. .125.	Ans. $\frac{1}{8}$.	8. .075.	Ans. $\frac{3}{40}$.
5. .625.	Ans. $\frac{5}{8}$.	9. .0125.	Ans. $\frac{1}{80}$.

CASE II.

132. To reduce a common fraction to a decimal.

1. Reduce $\frac{3}{4}$ to a decimal.

SOLUTION.—$\frac{3}{4}$ equals $\frac{1}{4}$ of 3. 3 equals 30 tenths, and $\frac{1}{4}$ of 30 tenths is 7 tenths and 2 tenths remaining. 2 tenths equals 20 hundredths, and $\frac{1}{4}$ of 20 hundredths is 5 hundredths; hence $\frac{3}{4} = .75$. From this we have the following

OPERATION.

$\frac{3}{4} = \frac{1}{4}$ of 3 =

4)3.00

.75

RULE.—1. *Annex ciphers to the numerator and divide by the denominator.*

2. *Point off as many places in the quotient as there are ciphers annexed.*

Reduce the following common fractions to decimals.

2. $\frac{1}{4}$.	Ans. .25.	7. $\frac{7}{16}$.	Ans. .4375.
3. $\frac{1}{8}$.	Ans. .125.	8. $\frac{9}{16}$.	Ans. .5625.
4. $\frac{5}{8}$.		9. $\frac{11}{25}$.	
5. $\frac{7}{8}$.		10. $\frac{13}{25}$.	
6. $\frac{5}{16}$.	Ans. .3125.	11. $\frac{15}{64}$.	

ADDITION OF DECIMALS.

133. ***Addition of Decimals*** is the process of finding the sum of two or more decimals.

1. What is the sum of 7.5, 18.25, 21.36 and 47.45?

SOLUTION.—We write the numbers so that figures of the same order shall stand in the same column, and commence at the right to add. 5 hundredths, plus 6 hundredths, plus 5 hundredths, equal 16 hundredths, which equal 1 tenth and 6 hundredths; we write the 6 hundredths, and add the 1 tenth to the next sum. 4 tenths, plus 3 tenths, plus 2 tenths, plus 5 tenths, are 14 tenths, and the 1 tenth added are 15 tenths, which equals 1 unit and 5 tenths; we write the 5 tenths, and add the 1 unit to the sum of the units, etc.

OPERATION.

7.5

18.25

21.36

47.45

94.56

RULE.—1. *Write the numbers so that units of the same order shall stand in the same column.*

2. *Add, as in whole numbers,* placing the decimal point *in its proper place in the sum.*

2. Find the sum of 12.05, 33.24, 47.62, 96.47.

3. Find the sum of 76.24, 89.45, 36.40, 85.75.

4. Find the sum of 79.76, 85.08, 95.42, 237.675.

5. Add 18.79, 147.072, 856.709, 185.8761, 397.05784.

6. Add 59.874, 435.095, 672.328, 976.309, 8467.500843.

7. Add together 9 and 7 tenths, 41 and 8 hundredths, 75 and 54 hundredths, 128 and 187 thousandths.

Ans. 254.507.

8. Add together 76 and 49 hundredths, 127 and 49 thousandths, 496 and 167 thousandths, 985 and 98 ten-thousandths, and 99 and 99 hundred-thousandths.

SUBTRACTION OF DECIMALS.

134. ***Subtraction of Decimals*** is the process of finding the difference between two decimals.

1. From 67.35 take 42.63.

SOLUTION.—We write the numbers so that figures of the same order stand in the same column, and begin at the right to subtract. 3 hundredths from 5 hundredths leave 2 hundredths; 6 tenths we cannot subtract from 3 tenths; we therefore take 1 unit from the 7 units, which with 3 tenths equal 13 tenths; then 6 tenths from 13 tenths leave 7 tenths, etc.	OPERATION 67.35 42.63 24.72

RULE.—1. *Write the smaller number under the greater, so that figures of the same order stand in the same column.*

2. *Subtract as in simple numbers, and place the decimal point in its proper place in the difference.*

2. From 63.72 take 25.81.

3. From 96.32 take 73.15.

4. From 123.16 take 75.84.

5. From 247.125 take 167.183.

6. From 1 and 1 tenth take 1 tenth and 1 thousandth.

7. From 2 and 2 hundredths take 2 tenths and 2 thousandths.

8. *From* 3 tenths take 3 ten-thousandths.

9. *From* 7 take 7 tenths and 707 millionths.

MULTIPLICATION OF DECIMALS.

135. ***Multiplication of Decimals*** is the process of multiplying when one or both terms are decimals.

1. Multiply 7.23 by .46.

SOLUTION 1.—Multiplying as in whole numbers, we have 33258; now, if the multiplicand alone were hundredths, the product would be one-hundredth of this, or 332.58; but since the multiplier is also hundredths, the product is one-hundredth of 332.58, which, by moving the decimal point two places to the left, becomes 3.3258.

OPERATION.

7.23
.46
4338
2892
3.3258

SOLUTION 2.—$7.23 \times .46 = \frac{723}{100} \times \frac{46}{100} = \frac{723 \times 46}{10000} = \frac{1}{10000} \times 33258 = 3.3258$. From either of these solutions we derive the following

RULE.—*Multiply as in whole numbers, and point off as many decimal places in the product as there are in both multiplier and multiplicand, prefixing ciphers when necessary.*

2. Multiply 15.17 by .18.
3. Multiply 26.18 by .25.
4. Multiply 53.46 by .35.
5. Multiply 67.38 by 1.26.
6. Multiply 138.25 by 2.47.
7. Multiply 466.72 by 5.29.
8. Multiply 407.03 by 7.35.
9. Multiply 620.75 by 12.36.
10. Multiply 725.82 by 23.08.
11. Multiply .00723 by .0317.
12. Multiply 1.0309 by .00321.

DIVISION OF DECIMALS.

136. ***Division of Decimals*** is the process of dividing when one or both terms are decimals.

1. Divide 7.8315 by 2.27.

SOLUTION.—If we divide as in whole numbers, we obtain a quotient of 345; now, since the dividend is the product of the divisor and quotient, the number of decimal places in the dividend must equal the number in the divisor and quotient; hence, the number of decimal places in the quotient must equal the number of decimal places in the dividend diminished by the number in the divisor; hence, there should be *four* minus *two*, or *two* decimal places in the quotient, therefore the quotient is 3.45.

OPERATION.

2.27)7.8315(3.45 Ans

6 81	
1 021	
908	
1135	
1135	

SOLUTION 2.—$7.8315 \div 2.27 = \frac{78315}{10000} \div \frac{227}{100} = \frac{78315}{10000} \times \frac{100}{227} = \frac{78315}{100 \times 227} = \frac{1}{100} \times \frac{78315}{227} = \frac{1}{100} \times 345 = 3.45$. From either of these solutions we derive the following

RULE.—*Divide as in whole numbers, and point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds the number in the divisor.*

NOTE 1.—When there are not as many decimal places in the dividend as in the divisor, annex ciphers to make the number of places equal.

2. When the number of figures in the quotient is less than the excess of the decimal places in the dividend over those in the divisor, ciphers must be prefixed to the quotient.

2. Divide 14.1372 by 4.5. Ans. 3.1416.
3. Divide 196.1875 by 10.75. Ans. 18.25.
4. Divide 25.1328 by 8. Ans. 3.1416.
5. Divide 65.9736 by 3.1416.
6. Divide 2450.448 by .5236.
7. Divide 2748.9 by .7854.
8. Divide 127.328 by .07958.
9. Divide 15.90435 by 20.25.
10. Divide 352.0625 by 32.75.

PRACTICAL PROBLEMS.

1. What cost 43.45 acres of land at $38.50 an acre?
Ans. $1672.825.

2. What cost 57.75 tons of hay at $12.25 a ton?
Ans. $707.4375.

3. If 31.25 yards of muslin cost $7.8125; how much is that a yard? Ans. $0.25.

4. A man sold 35.25 pounds of butter for $5.875; how much is that a pound? Ans. $0.166+.

5. There are 7.92 inches in a link; how many inches in 990 links? Ans. 7840.8 in.

6. There are 31.5 gallons in a barrel; how many barrels in 2756.25 gallons? Ans. 87.5 barrels.

7. If 14.5 yards of cloth cost $68.875, how much is that a yard? Ans. $4.75.

8. If a man walk 112·1184 miles in 9.16 days, how many miles does he walk each day? Ans. 12.24 miles.

9. How many yards of cloth at $4.28 a yard, can a person buy for $44.9828? Ans. $10.51.

10. What is the value of 54.6 multiplied by 80.5, and the product divided by 2? Ans. 2197.65.

11. The circumference of a water-wheel is 64 feet, and the diameter equals this divided by 3.1416; required the diameter? Ans. 20.3718 feet.

12. If 25.5 yards of cloth cost 195.375, how much will 45.25 yards cost? Ans. $346.695+.

13. If an imperial gallon contains 277.274 cubic inches, how many cubic inches in 328.55 gallons?

Ans. 91098.3727.

14. A gallon of distilled water weighs 8.33888 pounds; how many gallons in 1000 pounds of such water?

Ans. 119.92+.

15. A cubic inch of water weighs 252.458 grains; how many cubic inches in 157786.25 grains?

Ans. 625cu. in.

16. A drew 41.25 barrels, of 31.5 gallons each, from a cistern containing 2000 gallons; how much remained?

Ans. 700.625.

17. A bought 78.25 acres of land at $128.50 an acre, and sold it for $9781.25; what was the loss on each *acre?* Ans. $3.50.

REVIEW OF FUNDAMENTAL RULES.

HISTORICAL, GEOGRAPHICAL, ETC. PROBLEMS.

SUGGESTION.—The teacher should explain the nature of the facts presented, and require the pupils to remember some of the more important numbers and dates.

PROBLEMS
on Battles of the Revolution.

1. At the battle of Lexington, the Americans lost 90 men, the British 190 more; how many did the British lose?

2. At the battle of Bunker Hill, the Americans had 1500 men, the British 1500 more; how many had the British?

3. In this battle the Americans lost 450 men, the British 604 more; how many did the British lose?

4. At the battle of Long Island, the British lost 367 men, the Americans 1233 more; how many did the Americans lose?

5. At the battle of Trenton, the British lost 45 in killed and wounded, and 1000 prisoners; what was their loss?

6. In the battle of Brandywine, the British lost 800 men, and the Americans 450 more; how many did the Americans lose?

7. In the battle of Germantown, the British lost 600 men, and the Americans lost 600 more; how many did the Americans lose?

8. At the battle of Bennington, the Americans lost about 100, and the British 600 more; required the British loss.

9. At the battle of Monmouth, the Americans lost 70 in killed, and the British 230 more; required the British loss.

10. In taking Stony Point, Gen. Wayne lost 15 killed and 83 wounded, and the British lost 500 more in killed, wounded, and prisoners; required the British loss.

11. At the battle of Sander's Creek, the British lost 325, and the Americans 675 more; what was the American loss?

12. At the battle of King's Mountain, the Americans lost 20 men, and the British 280 more; how many did the British lose?

13. At the battle of Guilford, the Americans lost 400 men, the British lost 100 more; required the British loss.

14. At Hobkirk's Hill, the British loss was about 258 men, and the Americans 8 more; how many did the latter lose?

15. At Ninety-Six, the Americans lost 51 in killed and wounded; the British lost 1 more than this in killed, and 283 more in wounded; required the British loss.

16. At the battle of Eutaw Springs, the Americans lost 555, and the British 138 more; required the British loss.

17. At Yorktown, Washington had 11,000 Americans and 5000 French, and the British had 2000 more than the French; what was the force on each side?

18. At Yorktown, the Americans lost about 75 killed, and 225 wounded; the British lost 156 killed, 170 more than this wounded, and 70 missing; what was the loss on each side, not including prisoners?

PROBLEMS IN AMERICAN HISTORY.

1. America was discovered by Columbus in 1492, and Jamestown was settled in 1607; what was the difference of time?

2. Plymouth was settled in 1620; how long was that after America was discovered, and how long after the settlement of Jamestown?

3 The battle of Lexington was fought in 1775; how long was that after the settlement of Plymouth?

4 The Declaration of Independence was made in

1776; how long was that after the settlement of James town?

5. The surrender of Burgoyne took place in 1777; how long was that after the discovery of America?

6. The Inauguration of Washington took place in 1789; how long was that after the battle of Bunker Hill, in 1775?

7. The battle of New Orleans took place in 1815; how long was that after the inauguration of Washington?

8. The frigate Constitution captured the British frigate Guerriere in 1812; how long was it after the Declaration of Independence?

9. Commodore Perry won his great naval victory in 1813; how long was that after the battle of Lexington?

10. General Jackson won his great victory at New Orleans in 1815; how long is it from then till the present?

11. General Packenham had 12000 men, and General Jackson 6000; how many more had the British than the Americans?

12. The British lost 1700 in killed and wounded, the Americans 13 men; what was the difference?

13. War with Mexico commenced in 1846; how long was that after the battle of Lexington?

14. At the battle of Palo Alto, General Taylor had 2300 men and the Mexicans 6000; what was the difference?

15. The battle of Buena Vista was fought in 1847; how long was this after the battle of Bunker Hill?

16. At this battle General Taylor had 4759 men, while Santa Anna had 20000 men; required the difference of the forces.

17. General Scott took the Mexican capital in 1847; *how long* is it from that time to the present?

PROBLEMS ON THE AREA OF STATES.

NEW ENGLAND STATES.

1. The area of Maine is 30000 square miles, and of New Hampshire 9280 square miles; how much larger is the former?

2. Vermont contains 9056 square miles, and Massachusetts 7800 square miles; how much larger is the former than the latter?

3. Rhode Island contains 1306 square miles, and Connecticut 4674 square miles; how much larger is Maine than both of these?

4. Which is larger, and how much, Maine or all the rest of the New England States? Which is larger, and how much, New Hampshire and Vermont together, or Massachusetts and Connecticut together?

MIDDLE STATES.

5. New York contains 47000 square miles, and New Jersey 8320 square miles; how much larger is the former?

6. Pennsylvania contains 46000 square miles, and Delaware 2120 square miles; how much larger is Pennsylvania?

7. Maryland contains 9356 square miles; how much larger is Maryland than New Jersey?

8. How much larger is Pennsylvania than New Jersey, Delaware, and Maryland all together?

9. How much larger are the Middle States than the New England States?

WESTERN STATES.

10. Ohio contains 39964 square miles, and Indiana 33809 square miles; how much larger is the former than the latter?

11. Michigan contains 56243 square miles, and Illinois 55405 square miles; how much larger is the former *than the latter?*

12. How much larger are Ohio and Michigan than Indiana and Illinois?

13. Wisconsin contains 53924 square miles, and Iowa 55045 square miles; how much larger is the latter than the former?

14. Missouri contains 67380 square miles, and Kentucky 37680 square miles; how much larger is Missouri?

15. Which would make the larger State, Wisconsin and Iowa, or Missouri and Kentucky?

16. California contains 189000 square miles, and Oregon 95000 square miles; which is the larger, and how much?

17. How much larger are the Western States given than the New England and Middle States together?

SOUTHERN STATES.

18. Virginia contains 41352 square miles, and West Virginia 20000 square miles; how much larger is Virginia than West Virginia?

19. North Carolina contains 45000 square miles, and South Carolina 24500 square miles; how much larger is North Carolina than South Carolina?

20. Georgia contains 58000 square miles, and Louisiana 46431 square miles; how much larger is Georgia than Louisiana?

21. Alabama contains 50722 square miles, and Mississippi 47156 square miles; how much larger is Alabama than Mississippi?

22. Arkansas contains 52198 square miles, and Tennessee 45600; which is the larger, and how much?

23. Florida contains 59628 square miles, and Texas 237321 square miles; how much larger is Texas than Florida?

24. Which is larger, and how much, Texas, or all the *above States* taken together?

BUSINESS PROBLEMS.

SUGGESTION.—Pupils will put these in the form of accounts, as on page 93.

1. A merchant sold a farmer 125 yards of calico, at 18 cents a yard, 150 yards of drilling, at 15 cents a yard, and bought of the farmer 225 bushels of oats, at 40 cents a bushel, and 90 bushels of rye, at $1.25 a bushel; which owes the other, and how much?

2. A mechanic sold a farmer a wagon for $56.50, two plows, at $7.50 each, and 6 wheel-barrows, at $5.25 each; and bought of the farmer 50 bushels of potatoes, at 75 cents a bushel, and 75 bushels of wheat, at 85 cents a bushel; which owes the other, and how much?

3. A farmer sold a merchant 4 cows, at $28.50 each, a yoke of oxen for $95, and 7 sheep, at $6.25 each; and took in payment 40 yards of carpet, at $2.25 a yard, 35 yards of cloth, at $3.25 a yard, and 58 yards of muslin, at 15 cents a yard; how much remains due?

4. A farmer bought of a mechanic, 2 wagons, at $76 each, 4 drags, at $6.50 each, 3 harrows, at $12.25 each; and sold him 45 bushels of apples, at 55 cents a bushel, 3 barrels of cider, at $5.25 a barrel, 28 bushels of corn, at 42 cents a bushel, and 3 cows, at $28.75 each; which owes the other, and how much?

5. A mechanic bought of a merchant

28 pounds of sugar, at 18cts. a pound,
36 pounds of rice, at 17cts. a pound,
45 yards of muslin, at 18cts. a yard,
28 yards of cloth, at $5.25 a yard,
37 barrels of flour, at $7.25 a barrel;

And sold him

4 wagons, at $75 each,
6 wagon-racks, at $13.50 each,
2 mowing-machines, at $157 each,
3 ox-yokes, at $6.75 each;

Which owes the other, and how much?

SECTION VII.

DENOMINATE NUMBERS.

137. A ***Concrete Number*** is one which refers to some particular unit, as 2 books, 3 pounds, etc.

138. Concrete numbers are of two kinds; those in which the unit is *natural*, and those in which it is *artificial.*

139. A ***Denominate Number*** is a concrete number in which the unit is artificial, as 3 pounds, 4 yards, 5 minutes, etc.

140. ***Reduction*** is the process of changing a number from one denomination to another without changing its value.

141. ***Reduction Descending*** is the process of reducing from a higher to a lower denomination.

142. ***Reduction Ascending*** is the process of reducing from a lower to a higher denomination.

ENGLISH MONEY.

143. ***English,*** or ***Sterling Money,*** is the money of England.

TABLE.

4 farthings (far., or qr.)	equal	1 penny,	d.
12 pence	"	1 shilling,	s.
20 shillings	"	1 pound,*	£.
21 shillings	"	1 guinea.	g.

MENTAL EXERCISES.—Repeat the table of English Money.
How many far. in 2d.? in 3d.? in 6d.? in 8d.?
How many pence in 12far.? in 16far.? in 20far.? in 28far.?
How many pence in 2s.? in 3s.? in 5s.? in 6s.?
How many far. in 1s.? in 2s.? in 3s.? in 5s.?

---

* *The £ coined in gold is called a sovereign. Its value is $4.8665. A five-shilling piece in silver is called a crown. A two-and-a-half-shilling piece in silver is called a half-crown.*

REDUCTION DESCENDING.

1. How many farthings in 8 pence and 3 farthings?

SOLUTION.—In one penny there are 4 farthings, hence 4 times the number of pence equal the number of farthings; 4 times 8 are 32, and 32far. plus the 3far. equal 35 farthings.

OPERATION.

d.	far.
8	3
4	
32	
3	
35far.	Ans.

2. How many farthings in 14d. 2far.? Ans. 58far.
3. How many pence in 15s. 9d.? Ans. 189d.
4. How many shillings in £23 10s.? Ans. 470s.
5. How many farthings in 23s. 10d. 3far.?
6. How many pence in £32 19s. 3d.?

REDUCTION ASCENDING.

1. How many shillings, pence, and farthings, in 1487 farthings?

SOLUTION.—There are 4 farthings in one penny, hence in 1487far. there are as many pence as 4 is contained times in 1487, which are 371 pence, and 3far. remaining. There are 12 pence in one shilling, and in 371 pence there are as many shillings as 12 is contained times in 371, which are 30 shillings, and 11 pence remaining. Hence, 1485far. equal 30s., 11d., 3far.

OPERATION.

4)1487
12)371-3far.
30-11d.

2. How many shillings, pence, and farthings in 989 farthings? Ans. 20s. 7d. 1far.
3. Reduce 2676 farthings to shillings and pence. Ans. 55s. 9d.
4. How many pounds in 3178 farthings? Ans. £3 6s. 2d. 2far.
5. How many pounds in 9761 pence?
6. *How many* guineas in 17654 farthings?

TROY WEIGHT.

144. ***Troy Weight*** is used in weighing gold, silver, jewels, etc.

TABLE.

24 grains (gr.)	equal	1 pennyweight,	pwt.
20 pennyweights	"	1 ounce,	oz.
12 ounces	"	1 pound,	lb.

MENTAL EXERCISES.—How many oz. in 2lb.? in 3lb.? in 5lb.? How many lb. in 36oz.? in 48oz.? in 60oz.? in 1pwt.? in 2pwt.? How many pwt. in 2oz.? in 3oz.? in 4oz.? in 48gr.? in 72gr.?

1. How many grains in 15oz. 16pwt. 13gr.?

oz.	pwt.	gr.
15	16	13
20		
300		
16		
316 pwt.		
24		
1264		
632		
7584		
13		
7597 grs. Ans.		

2. How many pennyweights in 74597 gr.?

gr.
24)7597
2|0)31|6 — 13gr.
15 — 16pwt.
Ans. 15oz. 16pwt. 13gr.

NOTE.—In dividing by large numbers, like 24, we, of course, divide by Long Division. We indicated the result above.

How many

3. Grains in 6pwt. 12gr.? Ans. 156gr.
4. Pounds, oz., and pwt. in 963pwt.? Ans. 4oz. 3pwt.
5. Grains in 3oz. 11pwt. 14gr.?
6. Oz., pwt., and gr. in 5170gr.?
7. Pounds, oz., etc., in 15786gr.?

APOTHECARIES WEIGHT..

145. ***Apothecaries Weight*** is used in mixing medicines. Medicines are bought and sold by Avoirdupois Weight.

TABLE.

20 grains (gr.) . .	equal	1 scruple, ...	℈.
3 scruples . . .	"	1 dram, ...	ʒ.
8 drams	"	1 ounce, ...	℥.
12 ounces	"	1 pound, ...	℔.

MENTAL EXERCISES.—1. How many grs. in 2 scruples? in 3 scruples? in 4 scruples.

2. How many scruples in 40gr.? in 60gr.? in 80gr.? in 160gr.?

3. How many scruples in 2 drams? in 4 drams? in 2 ounces? in 4 ounces?

4. How many drams in 2 ounces? in 1℔? in 12℥? in 36℥? in 120gr.?

1. How many grains in 15ʒ 2℈ 12gr.?

OPERATION.

ʒ	℈	gr.
15	2	12
3		
47		
20		

952gr. Ans.

NOTE.—When convenient, add in the numbers as we multiply.

2. How many drams in 952 grains?

OPERATION.

2|0)95|2

3)47 — 12gr.

15 — 2℈

Ans. 15ʒ 2℈ 12gr

How many

1. Drams in 7℔. 5℥? Ans. 712ʒ.
2. Pounds and ounces in 239℥? Ans. 19℔. 11℥.
3. Scruples in 1 ℔. 8℥ 5ʒ 2℈? Ans. 5681℈.
4. Pounds, etc., in 92375gr.? Ans. 16℔. 3ʒ 1℈ 15gr.

AVOIRDUPOIS WEIGHT.

146. ***Avoirdupois Weight*** is used for weighing every thing except jewels, precious metals, etc.

TABLE.

16 drams (dr.)	equal	1 ounce,	oz.
16 ounces	"	1 pound,	lb.
25 pounds	"	1 quarter,	qr.
4 quarters	"	1 hundred-weight,	cwt.
20 hundred-weight	"	1 ton,	T.

MENTAL EXERCISES.—Ask mental questions upon this and the following tables, similar to those suggested under the previous tables.

1. How many drams in 12lb. 13oz. 9dr ?

OPERATION.

lb.	oz.	dr.
12	13	9
16		
72		
12		
13		
205 oz., etc.		

2. How many pounds in 3289 drams?

OPERATION.

drams.

16)3289

16)205 — 9dr.

12 — 13oz.

Ans. 12lb. 13oz. 9dr.

How many

3 Drams in 17lb. 12oz. 11dr.?
4 Ounces in 3qr 15lb. 13oz.?
5 Pounds in 5cwt. 3qr. 16lb.?
6. Pounds in 15675 drams?
7. Quarters in 27392 ounces?
8. Drams in 20cwt. 3qr. 14lb. 11oz. 9dr.?
9 Hundred-weight in 17896754 drams?

WINE MEASURE.

147. ***Wine Measure*** is used for measuring nearly all kinds of liquids.

TABLE.

4 gills (gi.)	equal	1 pint,	pt.
2 pints	"	1 quart,	qt.
4 quarts	"	1 gallon,	gal.

NOTE.—The *wine gallon* contains 231 cubic inches, while the *beer gallon*, used in measuring beer, and sometimes milk, contains 282 cu. in. In the old tables were given 31½ gals. = 1 barrel; 63 gals. = 1 hogshead; 2 hogsheads = 1 pipe; 2 pipes = 1 tun. These are not measures, however, but vessels of variable capacity.

How many

1. Pints in 4gal. 3qt. 1pt.?
2. Gallons in 976 pints?
3. Gills in 17gal. 2qt. 3gi.?
4. Gallons in 1763 gills?

DRY MEASURE.

148. ***Dry Measure*** is used in measuring dry substances, as grain, fruit, salt, coal, etc.

TABLE.

2 pints (pt.) . . .	equal	1 quart,	qt.
8 quarts	"	1 peck,	pk.
4 pecks	"	1 bushel,	bu.

NOTE.—A *chaldron* of 36 bushels is sometimes used for measuring coal.

How many

1. Pints in 3pk. 6qt. 1pt. of berries?
2. Bushels in 314 quarts of clover seed?
3. Bushels in 3157 pints of cranberries?
4. What cost 4 bushels of berries at 2 cents a pint?

APOTHECARIES' FLUID MEASURE.

149. ***Apothecaries' Fluid Measure*** is used for measuring liquids in preparing medical prescriptions.

TABLE.

60 minims (♏)	. equal	1 fluidrachm,	f℥.
8 fluidrachms	. "	1 fluidounce,	f℥.
16 fluidounces	. "	1 pint,	O.
8 pints	"	1 gallon,	Cong.

NOTE.—O is the initial of *octans*, the Latin for one-eighth, the pint being ⅛ of a gallon. Cong. is the abbreviation of *congiarium*, the Latin for gallon.

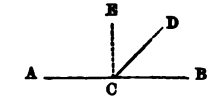
How many

1. Minims in 2O. 5f℥?
2. Pints in 8000 minims?
3. Fluidounces in 3Cong. 7O. 5f℥?
4. Gallons in 78561 minims?

MEASURE OF LENGTH.

150. ***Measure of Length,*** or ***Long Measure,*** is used for measuring length, breadth, height, distances, etc.

1. A ***Line*** is that which has length without breadth or thickness.



2. An ***Angle*** is the opening between two lines which diverge from a common point. Thus, ACD and DCB are angles.

3. A ***Right Angle*** is formed by one line perpendicular to another, as ACE or ECB.

TABLE.

12 inches (in.) . . .	equal	1 foot,	ft.
3 feet	"	1 yard,	yd.
5½ yards	"	1 rod,	rd.
40 rods	"	1 furlong,	fur.
8 furlongs	"	1 mile,	mi.
3 miles	"	1 league,	l.

NOTE.—*Cloth Measure* is not now used. Cloth, muslin, etc. are bought *by the yard, half-yard,* eighth, etc.

1. How many feet in 12rd. 3yd. 2ft.?

OPERATION.

rd. yd. ft.
12 3 2
5½
63
6
69yd.
3
209ft. Ans.

NOTE.—We multiply by 5, and add to the product the 3 yds., and then multiplying by ½, we have 69 yd.

2. How many rods in 209 ft.?

OPERATION.

feet.
3)209
5½)69ft.
2 2
11)138
12—6 halves=3yd.

Ans. 12rd. 3yd. 2ft.

NOTE.—To divide by 5½, we reduce both to halves, then the remainder is *halves*, which we reduce to *wholes*, by dividing by 2.

How many

3. Feet in 16rd. 5yd. 2ft.? Ans. 281ft.
4. Inches in 17yd. 1ft. 11in.? Ans. 635in.
5. Yards in 3146 inches? Ans. 87yds. etc.
6. Rods in 6547 inches?
7. Feet in 7fur. 32rd. 4yd.?
8. Furlongs in 4389 feet?
9. Miles in 19280 feet?
10. Inches in 2m. 6fur. 4rd. 8in.?
11. How many inches from New York to Philadelphia, if the distance is 96 miles?

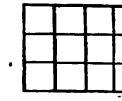
SURFACE OR SQUARE MEASURE.

151. ***Surface*** or ***Square Measure*** is used in measuring surfaces, as land, boards, etc.

1. A ***Surface*** is that which has length and breadth without thickness.

2. A ***Square*** is a surface which has four equal *sides and four right* angles, as in the margin.

3. A ***Rectangle*** is a surface which has four sides and four right angles. A slate, a door, the sides of a room, etc., are examples of rectangles.



4. The ***Area*** of a surface is expressed by the number of times it contains a small square as a *unit of measure.*

5. *The area of a square or rectangle is equal to the length multiplied by the breadth.* For, in the rectangle above, the whole number of little squares is equal to the number in each row multiplied by the number of rows: that is, 4×3 which equals 12, which is the same as the number of units in length multiplied by the number in breadth.

TABLE.

144 square inches (sq. in.)	equal	1 square foot,	sq. ft.
9 square feet	"	1 square yard,	sq. yd.
30¼ square yards . . .	"	1 perch, or sq. rod,	P.
40 perches	"	1 rood,	R.
4 roods	"	1 acre,	A.
640 acres	"	1 square mile,	sq. mi.

1. How many square feet in 28P. 18sq. yd. 5 sq. ft.?

OPERATION.

P.	sq. yd.	sq. ft.
28	18	5
30¼		
858		
7		
865sq. yd.		
9		
7790sq. ft.		

NOTE.—We multiplied by 30, added in the 18 sq. yds., and then *multiplied by ¼* and took the sum.

2. How many perches are there in 7790sq. ft.?

OPERATION

sq. ft.

9)7790

30¼)865 — 5sq. ft.

4 4

121)3460

28 — 72 fourths, or 18sq. yd.

Ans. 27P. 18sq. yd. 5sq. ft.

NOTE.—To divide by 30¼ we reduce both divisor and dividend to 4ths, and then divide; the remainder is 72 fourths, or 18 sq. yd.

How many

3. Square inches in 2sq. yd. 3sq. ft.?
4. Square feet in 2R. 13P. 16sq. yd.?
5. Perches in 8765 square feet?
6. Acres in 1997 perches? Ans. 12A. 1R. 37P.
7. Perches in 29A. 3R. 19P.? Ans. 4779P.
8. Acres in 89763 square yards?

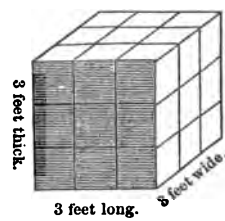
Ans. 18A. 2R. 7P., etc.

CUBIC OR SOLID MEASURE.

152. ***Cubic*** or ***Solid Measure*** is used in measuring things which have length, breadth, and thickness.

1. A ***Volume*** is that which has length, breadth, and thickness. A volume is also called a *solid.*

2. A ***Cube*** is a volume bounded by six equal squares. A ***Rectangular Volume*** is one bounded by rectangles. Cellars, boxes, rooms, etc., are examples of rectangular volumes.



3. The ***Contents*** of a volume are expressed by the number of times it contains a *cube* as a *unit of measure.*

The contents of a Cube or Rectangular Solid are equal to the product of the length, breadth, and height.

For in the volume above, the number of cubic units on the base is equal to the length multiplied by the breadth, and the whole number of cubic units equals the number on the base multiplied by the number of layers; hence the whole number equals $3 \times 3 \times 3 = 27$.

4. A *cord of wood* is a pile 8 feet long, 4 feet wide, and 4 feet high. A *cord foot* is a part of this pile, 1 foot long; it equals 16 cubic feet.


1 cord.

TABLE.

1728 cubic inches (cu. in.)	equal	1 cubic foot,	cu. ft.
27 cubic feet	"	1 cubic yard.	cu. yd
16 cubic feet	"	1 cord foot,	cd. ft.
8 cord feet, or 128 cubic feet, . . .	"	1 cord of wood,	Cd.
40 feet of round timber, or 50 feet of square timber,		1 ton,	tn.

How many

1. Cu. in. in 7cu. ft. 96cu. in.?
2. Cu. in. in 12cu. yd. 25cu. ft.?
3. Cu. ft. in 8469 cubic inches?
4. Cu. yd. in 60463 cubic inches?
5. Cords in 8192 cubic feet?

MEASURE OF TIME.

153. *Time* is the measure of duration.

TABLE.

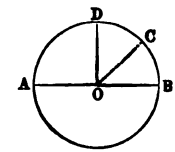
60 seconds (sec.) .	equal	1 minute,	min
60 minutes . . .	"	1 hour,	h.
24 hours	"	1 day,	da.
7 days	"	1 week,	wk.
4 weeks	"	1 month,	mo
52 weeks	"	1 year,	yr.
365 days	"	1 common year,	yr.
100 years	"	1 century,	cen.

How many

1. Minutes in 1 day? Ans. 1440.
2. Seconds in 1 day? Ans. 86400.
3. Hours in 1 year? Ans. 8760.
4. Minutes in 1 year? Ans. 525600.
5. *Hours in* 56780 seconds? Ans. 15h. etc
6. *Days in* 600000 seconds? Ans.

CIRCULAR MEASURE.

154. ***Circular Measure*** is used to measure angles and directions, latitude and longitude, etc.



1. A ***Circle*** is a figure bounded by a curve line, every point of which is equally distant from a point within, called the *centre.*

2. The ***Circumference*** is the bounding line; any part of the circumference, as BC, is an *arc;* AB is the *diameter,* and OC the *radius.*

3. For the purpose of measuring angles, the circumference is divided into 360 equal parts, called *degrees;* each degree into 60 equal parts, called *minutes;* each minute into 60 equal parts, called *seconds.*

4. Any angle at the centre, as COB, is measured by the arc BC included between its sides. A right angle is measured by 90 degrees; half a right angle, by 45 degrees, etc.

TABLE.

60 seconds (″)	. .	equal 1 minute,	. . ′
60 minutes	. . .	" 1 degree,	. . . °
30 degrees	. . .	" 1 sign	. . . . S.
12 signs, or 360°,	.	" 1 circumference,	C.

How many

1. Seconds in 24′ 32″? Ans. 1472″.
2. Seconds in 23° 24′ 15″? Ans. 84255″.
3. Minutes in 1472″? Ans. 24′ 32″.
4. Degrees in 84255″? Ans. 23° 24′ 15″.
5. What is the difference between the number of minutes in a day and the number of minutes in a circumference?
6. What is the difference between the number of seconds in a day and the number of seconds in a circumference?
7. If you study 6 hours a day, for 5 days in a week, how many minutes will you study in a week?

MISCELLANEOUS TABLES.

155. Counting.

12 units	equal	1 dozen.
12 dozen	"	1 gross.
12 gross	"	1 great gross.
20 units	"	1 score.

156. Paper.

24 sheets	equal	1 quire.
20 quires	"	1 ream.
2 reams	"	1 bundle.
5 bundles	"	1 bale.

157. Weight, Capacity, Length, etc.

56 pounds of rye or corn	equal	1 bushel.
60 pounds of wheat or clover seed	"	1 bushel.
60 pounds of beans or potatoes	"	1 bushel.
100 pounds of fish	"	1 quintal.
196 pounds of flour	"	1 barrel.
220 pounds of shad or salmon	"	1 barrel.
200 pounds of other fish	"	1 barrel.
200 pounds of beef or pork	"	1 barrel.
14 pounds (by English law)	"	1 stone.
8 bushels of wheat	"	1 quarter.
4 inches	"	1 hand.
9 inches	"	1 span.
22 inches (in Scripture)	"	1 cubit.

A knot or nautical mile is 6086.7 feet.

A surveyor's chain of 100 links is 4 rods long.

1. How many units in a gross?
2. How many pins in a great gross?
3. How many sheets in a ream? In a bundle?
4. How many sheets in a bale? In 12 bales?
5. How many bushels of rye will weigh as much as 14 bushels of wheat? Ans. 15bu.
6. How many bushels of beans will weigh as much as *30 bushels* of corn? Ans. 28bu.
7. *If Dr.* Windship lifts 3000 pounds, how many barrels of beef can he lift? Ans. 15 barrels.

MISCELLANEOUS PROBLEMS.

1. Reduce 967 pence to pounds.
2. Reduce 1840 pence to pounds.
3. Reduce 2480 farthings to shillings.
4. How many pounds in 8000 grains Troy?
5. How many pounds in 10000 ounces Avoirdupois?
6. Reduce £12 9s. 6d. to pence.
7. Reduce 8lb. 7oz. 13pwt. to pennyweights.
8. Reduce 12lb. 14oz. 15dr. to drams.
9. How many seconds in 24 hours, or one day?
10. How many pounds in 16cwt. 3qr. 13lb.?
11. How many tons in 9876 pounds?
12. How many tons in 165762 ounces?
13. Change 6ʒ 2ȝ 12gr. to grains.
14. Reduce 9ʒ 4ʒ 1ȝ 10gr. to grains.
15. How many pounds in 58763?
16. How many pounds in 76542ȝ?
17. Reduce 3m. 7fur. 4rd. 2yd. to yards.
18. Reduce 47692 feet to miles.
19. Reduce 1234560 inches to miles.
20. Adam died at the age of 930 years; how many seconds was this?
21. Methuselah died at the age of 969 years; how many seconds was this?
22. If the pulse beat 75 times a minute, how often does it beat in a day? Ans. 108000 times.
23. How long will it take to count a million, at the rate of a hundred a minute, working 12hrs. a day?
Ans. 13d. 10h. 40m.
24. If the distance around the earth is 25000 miles, how long will it take to walk the distance, walking 4 miles an hour? Ans. 260d. 10h.
25. If £1 equals $4.8665, what is the value of £5 in United States Money?

26. How many times will a clock that ticks seconds, tick in one day? Ans. 86400 times.

27. A little girl picked 2½ pecks of berries and sold them at 5 cents a pint; what did she receive? A. $2.00.

28. How many crayons are there in 25 boxes, if each box contains one gross? Ans. 3600.

29. How many vials, holding 2 gills each, can be filled from a gallon of brandy? Ans. 16.

30. If you are 10 years old, how many minutes have you lived, allowing 365¼ days to a year? A. 5259600 min.

31. How many doses of medicine, of 6 gr. each, can be made from 4 drams? Ans. 40.

32. If £1 equals $4.8665, required the value of £7 15s. in the money of the United States. Ans. $37.715.

33. If £2 equals $9.733, what is the value of $37.96 in English Money? Ans. £7 16s.+.

34. If 12 of Henry's peaches fill a quart measure, how many will there be in a bushel? Ans. 384.

35. How many square rods in a rectangular field 32 rods long and 12 rods wide? Ans. 384 sq. rods.

36. How many square feet in a board 18 feet long, and 2½ feet wide? Ans. 45 sq. feet.

37. How many cubic feet in a block of stone 6 feet long, 3 feet wide, and 2 feet thick? Ans. 36 cu. feet.

38. Required the value of a rectangular lot 36 rods long, and 20 rods wide, at $3 a square rod? Ans. $2160.

39. How many cords in a pile of wood 48 feet long, 4 feet wide, and 4 feet high? Ans. 6 cords.

40. How many cords in a pile of wood 16 feet long, 8 feet high, and 4 feet wide? Ans. 4 cords.

41. What must I pay for a pile of wood 24 feet long, 12 feet high, and 4 feet wide, at $1.50 a cord?

Ans. $13.50.

42. How much time is wasted by taking an hour's *nap each* afternoon, for 24 years of 365 days each?

Ans. 1 year.

43. When apples sell at 16 cents a half-peck; what are they worth a bushel? Ans. $1.28.

44. What will it cost to pave 75 sq. yards of walk at 50 cents a sq. foot? Ans. $337.50.

45. At 5 cents a half-pint, how much does a milkman receive for 25 gallons of cream? Ans. $20.00.

46. A grocer sold 8 bushels of chestnuts at 6 cents a quart; what did he receive for them? Ans. $15.36.

47. What will 12 pounds of drugs cost at the rate of 32 cents a dram? Ans. $983.04.

48. A grocer bought 132 eggs at 18 cents a dozen; what did they cost? Ans. $1.98.

49. How many steps of 3 feet each will a person take in walking 2½ miles? Ans. 4400.

50. A grocer bought 16 barrels of beef at 10½ cents a pound; what did it cost him? Ans. $336.

51. How much will 2 A. 20 P. of land cost at $2½ a perch? Ans. $850.

52. A man bought 32 reams of paper at 18¾ cents a quire; what was the cost? Ans. $120.

53. How many acres are there in a lot of land 160 rods long and 80 rods wide? Ans. 80 acres.

54. What cost 2 barrels of alcohol, each containing 31½ gallons, at 3½ cents a gill? Ans. $70.56.

55. How much will I get for 16 gross of pins at 1½ cents for each pin? Ans. $34.56.

56. If Dr. Davis use 2ʒ. 5ʒ. 2℈. of drugs daily, how much will he use in a week? Ans. 1℔. 6ʒ. 7ʒ. 2℈.

57. If blackberries are worth $3.20 a bushel, what are they worth a quart? Ans. 10 cents.

58. How many half-pint bottles will two gallons of ink fill? Ans. 32

59. What will 20 gross of lead-pencils cost at 62½ cents a dozen? Ans. $150.

60. How many quart baskets will 2 bu. 2 qts. of strawberries fill? Ans. 66.

61. How many ounces of calomel will it take to make 384 pills of 5 grains each? Ans. 4 oz.

62. Dr. Hess made calomel pills of 5 grains each, in all 2℥. 23. 1℈.; how many pills did he make? Ans. 220.

63. At $1.50 a bushel, what will a farmer receive for 2400 pounds of clover-seed? Ans. $60.

64. How many panels, of 10 feet, of fence will enclose a field which is 40 rods long and 30 rods wide? Ans. 231.

65. What will 2 bu. 3 pk. 6 qt. of shellbarks cost at 12 cents a quart? Ans. $11.28.

66. How many sheets of paper in 2½ reams and 3¼ quires? Ans. 1284.

67. A farmer put up 1¾ miles of fence at $1½ a rod; what did it cost? Ans. $800.

68. How many pounds in a load of corn which, at 75 cents a bushel, cost $32.25? Ans. 2408 lbs.

69. How many bushels of grapes at 15 cents a quart can be bought for $8.80? Ans. 1 bu. 3 pk. 2 qt. 1½ pt.

70. How many miles of fence, at $1.50 a rod, can be put up for $1200. Ans. 2 miles, 160 rods.

71. A man sold 2520 lbs of wheat at $1.87½ per bushel; what did he get for the whole? Ans. $78.75.

72. What will 36 packages of paper cost at 16⅔ cents a quire, if each package contains 2 reams? Ans. $240.

ADDITION OF DENOMINATE NUMBERS.

158. ***Addition of Denominate Numbers*** is the process of finding the sum of two or more denominate numbers of the same kind of quantity.

1. Find the sum of £8 7s. 5d.; £9 8s. 6d.; £7 14s. 9d

SOLUTION.—We write the numbers so that units of the same kind shall stand in the same column, and begin at the right to add. 9d plus 6d. plus 5d. equal 20d., which, by reduction, we find equal 1s. and 8d. We write the 8d. under the *pence column*, and reserve the 1s. to add to the *column of shillings*. 1s. plus 14s. plus 8s.

OPERATION.

£	s.	d.
8	7	5
9	8	6
7	14	9
25	10	8

plus 7s. equal 30s., which, by reduction, we find equal £1 and 10s. We write the 10s. in shillings column, and add the £1 to the column of pounds, etc. Hence the following

RULE.—1. *Write the numbers so that units of the same name stand in the same column, and commence at the right to add.*

2. *Add as in simple numbers, reduce by division the sum of each column to the next higher denomination, write the remainder under the column added, and add the quotient to the next column.*

3. *Proceed in the same manner with all the columns to the last, under which write the entire sum.*

PROOF.—The same as in simple numbers.

(2.)

£	s.	d.
24	12	6
25	13	9
17	18	10

(3.)

£	s.	d.
25	16	8
17	13	9
14	17	11

(4.)

£	s.	d.
123	14	6
137	18	10
246	19	11

(5.)

lb.	oz.	pwt.
17	9	16
25	6	12
72	11	13
57	10	19

(6.)

lb.	oz.	pwt.
18	9	16
36	8	21
29	7	23
42	11	17

(7.)

lb.	oz.	pwt.
92	7	12
71	3	17
28	9	10
86	11	18

(8.)

cwt.	qr.	lb.	oz
20	3	12	11
16	2	16	12
17	0	22	20
19	1	18	19

(9.)

qr.	lb.	oz.	dr.
12	16	12	11
13	23	9	10
14	24	14	15
15	16	15	8

(10.)

rd.	yd.	ft.	in
17	4	2	6
21	2	1	7
23	3	0	8
25	5	2	9

(11.)					(12.)			(13.)			
℔.	℥	ʒ	℈	gr.	gal.	qt.	pt.	L.	mi.	fur.	rd.
28	11	7	2	16	36	2	1	16	2	7	30
19	9	5	1	23	42	1	1	14	2	7	32
27	8	3	2	17	25	3	0	28	1	6	28
24	7	2	1	18	28	3	1	34	0	5	37

SUBTRACTION OF DENOMINATE NUMBERS.

159. ***Subtraction of Denominate Numbers*** is the process of finding the difference between two compound numbers of the same kind of measure.

1. From 10oz. 12pwt. 20gr. take 7oz. 15pwt. 16gr.

SOLUTION.—We write the subtrahend under the minuend, writing units of the same name in the same column, and commence at the lowest denomination to subtract. 16gr. subtracted from 20gr. leave 4grs., which we write under the grains. 15pwt. from 12pwt. we cannot take; we will therefore take 1oz. from the 10oz., leaving 9oz.; 1oz. equal 20pwt., which added to 12pwt. equal 32pwt.; 15pwt. subtracted from 32pwt. equal 17pwt., which we write under pwts. 7oz. from 9oz. (or since it will give the same result, we may add 1oz to 7oz. and say, 8oz. from 10oz.) leave 2oz. Hence the following

OPERATION.

oz.	pwt.	gr.
10	12	20
7	15	16
2	17	4

RULE.—1. *Write the subtrahend under the minuend, with units of the same denomination in the same column.*

2. *Commence at the lowest denomination, and subtract each number in the subtrahend from the corresponding number in the minuend.*

3. *If the number in the subtrahend exceeds the number in the minuend, add to the latter as many units of that denomination as make one of the next higher, and then subtract; add also, one to the next number in the subtrahend before subtracting.*

4. *Proceed in the same manner with each denomination to the last.*

PROOF.—The same as in simple numbers.

(2.)

£	s.	d.	far.
143	11	10	2
115	14	6	3
27	17	3	3

(3.)

£	s.	d.	far.
930	17	7	3
246	19	8	1

(4.)

lb.	oz.	pwt.	gr
16	10	16	18
13	11	17	15
2	10	19	3

(5.)

lb.	oz.	pwt.	gr.
125	8	14	20
96	9	10	23

(6.)

cwt.	qr.	lb.	oz.
112	3	17	12
37	1	10	13

(7.)

T.	cwt.	qr.	lb.	oz
236	13	2	18	12
127	11	4	22	10

(8.)

hhd.	gal.	qt.	pt.
128	27	0	1
106	30	2	1

(9.)

yr.	mo.	wk.	da.	h.
216	10	2	5	16
123	10	3	2	20

(10.)

sq.yd	sq.ft.	sq.in
226	0	120
134	5	130

(11.)

A.	R.	P.
426	1	30
207	3	35

(12.)

L.	mi.	fur.	rd.
16	2	7	30
14	2	7	32

(13.)

S.	°	′	″
25	20	30	40
20	30	40	50

14. A farmer had 200bu. of wheat, and sold 28bu. 2pk. 5qt. 1pt. to one man, and as much more to another; how much remained? Ans. 142bu. 2pk. 5qt.

15 A miner having 112lb. of gold sent his mother 17lb. 10oz. 15pwt. 20gr. and 3lb. 16pwt. less to his father; how much did he retain?

Ans. 79lb. 3oz. 4pwt. 8gr.

16. Subtract 16dol. 57cts. 5½ mills from $25 20cts. 7½ mills, and add 2 eagles and 25¾ dimes to the result.

Ans. 31dol. 20cts. 7 mills.

17. Add 96lb. 9oz. 10pwt. 23gr. to 125lb. 8oz. 14pwt. 20gr. and subtract the sum from the sum of 102lb. 11oz. 16pwt. and 256lb. 9oz. 19pwt.

MULTIPLICATION OF DENOMINATE NUMBERS

160. ***Multiplication of Denominate Numbers*** is the process of multiplying a denominate number by an abstract number.

1. Multiply £12 11s. 7d. by 8.

SOLUTION.—8 times 7d. are 56d., which by reduction we find is equal to 4s. and 8d. We write the 8 pence under the pence, and reserve the 4s. to add to the next product. 8 times 11s. are 88s., which added to the 4s. equal 92s., which we find by reduction equal £4 and 12s. 8 times £12 are £96, which added to £4 equal £100 Hence the following

OPERATION.

£	s.	d.
12	11	7
		8
100	12	8

RULE.—*Write the multiplier under the lowest denomination of the multiplicand, multiply as in simple numbers, reducing as in addition.*

PROOF.—The same as in simple numbers.

EXAMPLES FOR PRACTICE.

(2.)

cwt.	qr.	lb.	oz.
18	3	21	9
			5

(3.)

lb.	oz.	pwt.	gr.
16	8	15	17
			3

(4.)

M.	da.	h.	min.	sec.
50	10	20	30	40
				7

(5.)

£	s.	d.	far.
13	12	9	2
			8

(6.)

hhd.	gal.	qt.	pt.
21	35	3	1
			9

(7.)

lb	ʒ	ʒ	℈	gr.
12	8	7	2	20
				11

8. Multiply 12L. 2mi. 5fur. 32rd. by 5, by 6, by 7, by 8.

9. Multiply 23ch. 18bu. 2pk. 7qt. 1pt. by 4, by 5, by 9, by 10.

10. A farmer sold 5 loads of hay, each containing 15cwt. 3qr. 15lb.; how much did he sell?

Ans. 79cwt. 2qr.

11. Multiply 13yr. 10mo. 3wk. 5da. by 5, and that product by 3. Ans. 208yr. 7mo. 3wk. 5da.

12. If a man walk 17mi. 7fur. 20rd. in each of 21 days, how far will he walk in all?

Ans. 376mi. 5fur. 20rd.

13. If a farmer raise 60bu. 3pk. 6qt. 1pt. of grain on one acre, how much can he raise at the same rate on 48 acres? Ans. 2925bu. 3pk.

14. A owned 1000A. of land; he sold B 96A. 3R. 30P., and 4 times as much to C; how much remained?

Ans. 515A. 1R. 10P.

DIVISION OF DENOMINATE NUMBERS.

161. ***Division of Denominate Numbers*** is the process of dividing when one or both terms is a denominate number. There are two cases.

CASE I.

162. To divide a denominate number into equal parts.

1. Divide £103 7s. 6d. into 5 equal parts; that is, take $\frac{1}{5}$ of it.

SOLUTION.—$\frac{1}{5}$ of £103 is £20 and £3 remaining. £3 equal 60s., which added to 7s. equal 67s.; $\frac{1}{5}$ of 67s. is 13s. and 2s. remaining. 2s. equal 24d., which added to 6d. equal 30d.; $\frac{1}{5}$ of 30d. is 6d. Hence the following

OPERATION.

£	s.	d.
5)103	7	6
20	13	6

RULE.—1. *Begin at the highest denomination, and divide as in simple numbers.*

2. *If there is a remainder, reduce it to the next lower denomination, add to it the number of that denomination, and divide as before, and thus* continue to the last.

EXAMPLES FOR PRACTICE.

(2.)

£	s.	d.
4)61	18	4
15	9	7

(3.)

lb.	oz.	pwt.	gr.
6)76	10	14	12

(4.)

T.	cwt.	qr.	lb.
7)112	16	2	16
16	2	1	13

(5.)

cwt.	qr.	lb.	oz.	dr.
8)125	3	19	12	8

(6.)

hhd.	gal.	qt.	pt	gi
9)108	42	2	1	2

(7.)

mi.	fur.	rd.	yd.	ft.
11)120	7	33	3	2

(8.)

A.	R.	P.	sq. yd.
5)112	3	24	24

(9.)

bu.	pk.	qt.	pt.
9)1137	3	4	1

(10.)

lb.	oz.	pwt.	gr.
8)37	10	17	16

11. A miner divides 37lb. 10oz. 17pwt. 16gr. of gold among 9 sisters; how much does each receive?

Ans. 4lb. 2oz. 10pwt. $20\frac{4}{9}$gr.

12. A man walked 376mi. 6fur. 36rd. in 22 days; what was the average distance each day?

Ans. 17mi. 1fur. $1\frac{7}{11}$rd.

13. If 26 casks contain 21hhd. 11gal. 2qt. 1pt., what is the capacity of each cask? Ans. 51gal. 1qt. $\frac{17}{26}$pt.

Case II.

163. To divide a denominate number by a similar denominate number.

1. Divide £26 6s. 2d. by £4 15s. 8d.

Solution.—£26 6s. 2d. we find by reduction equal 6314 pence; £4 15s. 8d. equals 1148 pence; and dividing 6314d. by 1148d. we obtain a quotient of $5\frac{1}{2}$. From this solution we have the following

OPERATION.

£26 6s. 2d. = 6314d.

£4 15s. 8d. = 1148d.

1148)6314(5½, Ans.
6314

Rule.—Reduce both dividend and divisor to the lowest

denomination mentioned in either, and then divide as in simple numbers.

REMARK.—The division may also be made before the reduction to lower denomination; and this will be shorter when there is no remainder.

2. Divide £48 7s. 4d. by £6 11d. Ans. 8.

3. Divide 69bu. 3pk. 6qt. by 6bu. 3pk. 6qt.
Ans. $10\frac{3}{37}$.

4. Divide 80bu. 2pk. 4qt. by 13bu. 3pk. 5qt.
Ans. $5\frac{71}{89}$.

5. Divide 697lb. 7oz. 5dr. by 60lb. 10oz. 6dr.
Ans. $11\frac{1}{2}$.

6. A man travelled 3mi. 6fur. 36rd. 4yd. in one hour; in what time will he travel 247mi. 2fur. 30rd. 3yd.?
Ans. 64 hrs.

7. A drove of cattle ate 6T. 15cwt. 3qr. 12lb. of hay in a week; how long will 33T. 19cwt. 1qr. 10lb. last them? Ans. 5 weeks.

PROBLEMS IN TIME.

1. Washington was born Feb. 22d, 1732, and died Dec. 14th, 1799; what was his age?

SOLUTION.—We write the number of the year, month, and day of both periods, and subtract the one from the other, as is shown in the margin.

OPERATION.	yr.	mo.	da.
	1799	12	14
	1732	2	22
	67	9	22

2. John Adams was born the 19th of October, 1735, and died the 4th of July, 1826; required his age.

3. Thomas Jefferson was born April 2d, 1743, and died July 4th, 1826; what was his age?

4. James Madison was born March 16th, 1751. and died June 28th, 1836; required his age.

5. James Monroe was born April 28th, 1758, and died *July 4th, 1831;* required his age.

6. John Quincy Adams was born July 11th, 1767, and died Feb. 23d, 1848; what was his age?

7. Andrew Jackson was born March 15th, 1767, and died June 8th, 1845; required his age.

8. Martin Van Buren was born Dec. 5th, 1782, and died July 24th, 1862; required his age.

9. William Henry Harrison was born Feb. 9th, 1773 and died April 4th, 1841; required his age.

10. James K. Polk was born Nov. 2d, 1795, and died June 15th, 1849; required his age.

11. General Zachary Taylor was born Nov. 24th, 1784, and died July 9th, 1850; required his age.

12. How long has a note to run which is dated Dec. 30th, 1862, and made payable Jan. 16th, 1864?

Ans. 1yr. 16da.

13. The Revolution was commenced the 19th of April, 1775, and terminated January 20th, 1783; how long did it continue? Ans. 7yr. 9mo. 1da.

PROBLEMS IN LATITUDE AND LONGITUDE.

1. The latitude of Boston is 42° 21′ 23″ north; that of Charleston, 32° 46′ 33″ north; what is the difference of latitude?

2. The latitude of New York is 40° 24′ 40″ N.; that of New Orleans, 29° 57′ 30″ N.; what is the difference of latitude?

3. The latitude of Philadelphia is 39° 56′ 39″; that of Savannah is 32° 4′ 56″; what is the difference of latitude?

4. The latitude of Baltimore is 39° 17′ 23″; that of St. Louis is 38° 37′ 28″; what is the difference of latitude?

5. The latitude of the Cape of Good Hope is 30° 55′ 15″ S.; that of Cape Horn, 55° 58′ 30″; what is the *difference of* latitude?

SECTION VIII.

PERCENTAGE.

164. ***Percentage*** is a process of computation in which the basis of comparison is a hundred.

165. The term *per cent.* means *by* or *on a hundred;* thus, 5 per cent. of any thing means 5 of a hundred of it.

166. Hence, 1 per cent. of a number is $\frac{1}{100}$ of it; 2 per cent. is $\frac{2}{100}$ of it; 5 per cent. is $\frac{5}{100}$ of it, etc. It is also evident that 100 per cent. of a number is the whole of it.

167. ***The sign of Percentage*** is %, and is read *per cent.* The per cent. is also indicated by a common fraction or a decimal; thus, $5\% = \frac{5}{100} = .05$.

168. In percentage there are four quantities considered:

1. The *Base*, or number on which percentage is estimated.

2. The *Rate*, or number denoting how many of a hundred.

3. The *Percentage*, denoting how many of the basis.

4. The *Amount* or *Difference* of the basis and percentage.

RATE EXPRESSED BY A FRACTION.

2 per cent.	equals	.02, or $\frac{2}{100}$,	or $\frac{1}{50}$.
4 per cent.	"	.04, " $\frac{4}{100}$,	" $\frac{1}{25}$.
5 per cent.	"	.05, " $\frac{5}{100}$,	" $\frac{1}{20}$.
20 per cent.	"	.20, " $\frac{20}{100}$,	" $\frac{1}{5}$.
25 per cent.	"	.25, " $\frac{25}{100}$,	" $\frac{1}{4}$.
125 per cent.	"	1.25, " $\frac{125}{100}$,	" $\frac{5}{4}$.

$\frac{1}{2}$ per cent. is .00$\frac{1}{2}$, or .005; $\frac{1}{5}$ per cent. is .00$\frac{1}{5}$, or .002; *12$\frac{1}{2}$ per cent.* is .12$\frac{1}{2}$, or .125, etc.

EXERCISES.

Express decimally		Express in a common fraction	
1. 6%.	Ans. .06.	6. 6%.	Ans. $\frac{3}{50}$.
2. 12%.	Ans. .12.	7. 12%.	Ans. $\frac{3}{25}$.
3. $16\frac{2}{3}$%.	Ans. $.16\frac{2}{3}$.	8. 50%.	Ans. $\frac{1}{2}$.
4. 24%.	Ans. .24.	9. $12\frac{1}{2}$%.	Ans. $\frac{1}{8}$.
5. $33\frac{1}{3}$%.	Ans. $.33\frac{1}{3}$.	10. $16\frac{2}{3}$%.	Ans. $\frac{1}{6}$.

Express as a per cent.			
1. $\frac{1}{4}$.	Ans. 25%.	4. $\frac{1}{8}$.	Ans. $12\frac{1}{2}$%.
2. $\frac{1}{5}$.	Ans. 20%.	5. $\frac{1}{6}$.	Ans. $16\frac{2}{3}$%.
3. $\frac{1}{2}$.	Ans. 50%.	6. $\frac{2}{3}$.	Ans. $66\frac{2}{3}$%.

CASE I.

169. Given the base and rate, to find the percentage.

1. What is 5 per cent. of $250?

SOLUTION.—5 per cent. of $250 is $\frac{5}{100}$ of $250, or .05 times $250, which, by multiplying, we find to be $12.50. Hence the following

OPERATION.
$250
.05
$12.50

RULE.—*Multiply the base by the rate, expressed decimally.*

2. What is 5 per cent. of $280? Ans. $14.
3. What is 6 per cent. of $190? Ans. $11.40.
4. What is 7 per cent. of $240? Ans. $16.80
5. What is 8 per cent. of 125yd.? Ans 10yd.
6. What is 9 per cent. of 364lb.?
7. What is 10 per cent. of 982ft.?
8. What is 12 per cent. of 831in.?
9. What is $12\frac{1}{2}$ per cent. of 320oz.?
10. What is $16\frac{2}{3}$ per cent. of 630yds.?
11. What is 35 per cent. of 1286 miles?
12. What is 40 per cent. of 2467 pounds?
13. What is 75 per cent. of 3182 perches?

14. A man bought a cow for $30, and sold her at a gain of 25%; what did he gain? Ans. $7½.

15. A man bought a horse for $150, and sold him at a gain of 30%; how much did he gain? Ans. $45.

16. A lady bought 360 acres of land, and sold 12½% of it; how much did she retain? Ans. 315 acres.

17. A man bought a horse for $4250, and sold it at a gain of 5%; how much did he receive for it?

18. My salary is $1500 a year; I spend 25% of it the first half year, and 35% the second half; how much do I save in a year? Ans. $600.

Case II.

170. Given the percentage and rate, to find the base.

1. 60 is 5 per cent. of what number?

Solution.—If 5% of a number is 60, 1% of the number is ⅕ of 60, or 12, and 100% of the number, which is the whole number, is 100 times 12, or 1200.

Since this is equivalent to multiplying by 100 and dividing by 5, and this last is equivalent to dividing by .05, we have the following

OPERATION.

5% = 60

100% = 1200 Ans.

or,

60 ÷ .05 = 1200

Rule.—*Divide the percentage by the rate expressed decimally.*

Note.—For young pupils the analysis will be simpler than the solution by the rule.

2. 45 is 20 per cent. of what number? Ans. 225.
3. 75 is 25 per cent. of what number? Ans. 300.
4. 96 is 20 per cent. of what number?
5. 230 is 5 per cent. of what number?
6. 112lb. is 40 per cent. of what weight?
7. 456 acres are 30 per cent. of how many?
8. 237 cows are 25 per cent. of how many?

Ans. 948.

9. *157yd.* are 12½ per cent. of how many?

Ans. 1256.

10. 644 pigs are 35 per cent. of how many?

11. \$78.18 is $33\frac{1}{3}$ per cent. of how much?

Ans. \$234.54.

12. A man spends \$500 a year, which is 25% of his salary; what is his salary? Ans. \$2000.

13. A has 280 acres of land, and this is 35% of what B has; how much has B?

14. A sold 36 pigs, which is 8% of what he now has; how many had he before the sale?

15. A boy found \$15, which is 30% of what he then had; how much had he at first? Ans. \$35.

16. A man had \$13681.60 in a bank, and drew out 35% of it; how much did he draw out?

Ans. \$4788.56.

Case III.

171. Given the base and percentage, to find the rate.

1. 25 is what per cent. of 125?

SOLUTION.—125 is 100 per cent. of itself, and 25, which is $\frac{25}{125}$ of 125, is $\frac{25}{125}$ of 100 per cent., or $\frac{1}{5}$ of 100 per cent., which is 20 per cent. Hence the following

OPERATION.

$125 = 100\%$

$25 = \frac{25}{125} \times 100\%$

$= \frac{1}{5} \times 100\% = 20\%$

RULE.—*Take such a part of* 100 *as the percentage is of the base;* or, *multiply the percentage by* 100 *and divide by the base.*

2. 75 is what per cent. of 300? Ans. 25%.

3. 90 is what per cent. of 360? Ans. 25%.

4. 45 is what per cent. of 225? Ans. 20%.

5. 72 is what per cent. of 216? Ans. $33\frac{1}{3}$%.

6. 96 is what per cent. of 128?

7. 48 is what per cent. of 120?

8. 112 is what per cent. of 896?

9. A man had \$960, and lost \$240; what per cent. did *he lose?*

10. B lost $25, and then had $125; what per cent. of his money did he lose?

11. C sold 50 cows, which was 25 per cent. of the remainder; how many had he at first?

12. D gave his sister $180, and had $960 left; his money is now what per cent. of what he had at first?

SIMPLE INTEREST.

172. ***Interest*** is money charged for the use of money. It is estimated at a certain rate per cent. *per annum.*

173. The ***Principal*** is the sum on which interest is reckoned.

174. The ***Rate of Interest*** is the interest on 100 for one year.

175. The ***Time*** is how long the money is on interest.

176. The ***Amount*** is the sum of the principal and interest.

177. ***Simple Interest*** is interest upon the principal only. ***Compound Interest*** is interest upon the principal and interest.

178. ***Legal Interest*** is the rate established by law. ***Usury*** is a rate greater than the legal rate. The taking of usury is prohibited by law.

179. The quantities considered in Simple Interest are the *Principal*, *Rate*, *Time*, *Interest*, and *Amount*. There are four cases.

CASE I.

180. Given the principal, rate per cent., and time, to find the interest or amount.

First Method.

1. What is the interest of $2400, for 6yr. 7mo. 15da. at 7%?

SOLUTION.—By reduction we find that 6yr. 7mo. 15da. equal $6\frac{5}{8}$yr. If the interest of $1 for 1yr. is 7ct., the interest of $2400 for 1yr. is 2400 times 7ct., which is $168, and for $6\frac{5}{8}$yr. it is $6\frac{5}{8}$ times $168, which by multiplying we find is $1113. Hence the following

OPERATION.

$2400
.07
168.00
$6\frac{5}{8}$
$1113.00 Ans

RULE.—*Multiply the principal by the rate per cent., expressed decimally, and that product by the time expressed in years.*

Required the interest

2. Of $180 for 3yr. 6mo. at 7%. Ans. $44.10.
3. Of $470 for 7yr. 8mo. at 6%. Ans. $216.20.
4. Of $172 for 5yr. 9mo. at 5%. Ans. $49.45.
5. Of $480 for 5yr. 10mo. at 12%.
6. Of $1080 for 3yr. 7mo. 6da. at 5%.
7. Of $1260 for 2yr. 2mo. 12da. at 5%.
8. Of $1000 for 3yr. 8mo. 12da. at 10%.

Second Method.

181. The second method, called the "*Six per cent. method*," is perhaps the method most generally used by business men.

1. What is the interest of $240 for 2yr. 8mo. 12da. at 6%?

SOLUTION.—2yr. 8mo. equal 32mo. The interest of $1 for 12mo. is 6cts., and for 1mo. it is $\frac{1}{12}$ of 6cts., or $\frac{1}{2}$ct., and for 32mo. it is $32 \times \frac{1}{2} = 16$cts. Also, since the interest on $1 for 1mo., or 30da., is $\frac{1}{2}$ct. or 5 mills, for 1da. it is $\frac{1}{30}$ of 5 mills, or $\frac{1}{6}$ of a mill, and for 12da. it is $12 \times \frac{1}{6} = 2$ mills: hence the interest on $1 for 32mo. and 12da. is 16cts. plus 2 mills, *or $0.162. If* the interest on $1 is $0.162, on $240 it is 240 times *$0.162 which* equal $38.88. From this we have the following

OPERATION.

2yr. 8mo. = 32mo.
$32 \times \frac{1}{2} = \$0.16$
$12 \times \frac{1}{6} = .002$
$0.162
240
$38.88

RULE —1. *Call* ONE-HALF *of the number of months cents, and* ONE-SIXTH *of the number of days mills, and their sum will be the interest of one dollar, for the given time, at* 6 *per cent.*

2. *Multiply this by the principal, and the product will be the entire interest at* 6 *per cent. For any other rate, take as many sixths of it as that rate is of six.*

NOTE.—1. For 7% add $\frac{1}{6}$, for 8% add $\frac{2}{6}$, for 9% add $\frac{3}{6}$, for 5% subtract $\frac{1}{6}$, for 4% subtract $\frac{2}{6}$, etc.

2. When the time is brief, the rule of business men is as follows: "*Multiply dollars by days, and divide by* 60."

Required the interest

1. Of \$360 for 5yr. 6mo. 12da. at 6%.
Ans. \$119.52.
2. Of \$480 for 3yr. 8mo. 18da. at 6%.
Ans. \$107.04.
3. Of \$256 for 7yr. 4mo. 24da. at 6%.
Ans. \$113.66.
4. Of \$48.25 for 3yr. 6mo. 6da. at 6%.
Ans. \$10.18
5. Of \$50.50 for 6yr. 10mo. 18da. at 7%.
Ans. \$24.33.
6. Of \$28.25 for 5yr. 7mo. 24da. at 5%.
7. What is the amount of \$360 for 2yr. and 6mo. at 6 per cent.? Ans. \$414.
8. What is the amount of \$250 for 3yr. 8mo. 18da. at 6 per cent.? Ans. \$305.75.
9. What is the amount of \$620 for 5yr. 10mo. 24da. at 7 per cent.? Ans. \$876.06.
10. Mary's father put out \$500 on interest, at 10% at her birth; how much will she be worth when she is 21 years of age?
11. Required the difference between the interest and the amount of \$624 for 3yr. 8mo. 15da., at 5 per cent.

For other exercises under this rule solve the problems in the previous and following cases.

Third Method.

182. A method of computing interest by taking aliquot parts.

1. What is the interest of $2400 for 6yr. 7mo. 15da at 7%?

SOLUTION. — We find the interest for 1 yr. and then for 6yr., and then proceed thus: 7mo. = 6mo. plus 1mo., and since 6mo. equal ½ of a year, the interest for 6mo. is ½ of $168, which is $84; and since 1mo. is ⅙ of 6mo., the interest for 1mo. is ⅙ of $84, which is $14; and since 15 days is ½ of a month, the interest for 15 days is ½ of $14, which is $7; and the whole interest is the sum of these, which we find to be $1113. Hence the following

OPERATION.

$2400
.07
168.00 = Int. for 1yr.
6
1008.00 = Int for 6yr
6mo. = ½ yr. 84.00 = Int. for 6mo
1mo. = ⅙ of ½yr. 14.00 = Int. for 1mo.
15da. = ½mo. 7.00 = Int. for 15da.
1113.00, Ans.

RULE.—1. *Find the interest for the number of years, as by the first method.*

2. *Find the interest for the number of months by taking convenient fractional parts of one year's interest.*

3. *Find the interest for the number of days by taking fractional parts of one month's interest.*

Required the interest

2. Of $780 for 4yr. 8mo. at 6%. Ans. $218.40.
3. Of $960 for 7yr. 9mo. at 7%. Ans. $520.80.
4. Of $1260 for 3yr. 6mo. 15da. at 8%. Ans. $357.
5. Of $2480 for 5yr. 5mo. 10da. at 5%. Ans. $675.11.

NOTE.—Let the pupil solve by this method the problems under the first and second methods.

STOCKS AND DIVIDENDS.

183. The ***Stock*** of a company represents the money invested in its business by the stockholders or owners.

Stock is divided into equal parts, called *shares.*

184. ***A Dividend*** is a sum to be paid to the stockholders out of the gains of the company. It is divided according to the *par* value of stock held by them.

185. The ***Par Value*** of stock is its nominal value as fixed by the charter, or articles of agreement, of the company. It is usually $50 or $100 per share; although other sums are often agreed upon.

186. The ***Real Value*** of stock is what it will sell for.

187. ***Premium*** is how much its real value *exceeds* its par value.

188. ***Discount*** is how much its real value *is less* than its par value.

CASE I.

189. Given the stock and rate of dividend, to find the dividend.

1. A owns 50 shares of bank-stock, at $100 each; the bank declares a dividend of 6%; required A's dividend.

SOLUTION.—If one share is worth $100, 50 shares are worth 50 times $100, or $5000. Hence A's stock is worth $5000. His dividend is .06 times $5000, or $300.

OPERATION.

50
100
5000
.06
$300.00

RULE.—*Multiply the par value of the number of shares by the rate of dividend.*

2. *A man* owns 56 shares of stock, at $50 per share

par value; the company declares a 5% dividend; what is his dividend? Ans. $140.

3. A lady has 175 shares of stock, at $10 par per share; the company declares 8½% dividend; what is her dividend? Ans. $148.75.

4. I hold 250 shares of mining stock, at $20 par a share; the company has divided 5½%; what is my dividend? Ans. $275.

CASE II.

190. Given the par value and rate of premium or discount, to find the premium, discount, or real value.

1. A bought 25 shares of stock ($50), at 4% premium; required the premium and the real value.

SOLUTION.—The par value of 25 shares at $50 a share is 25 times $50, or $1250. The premium is .04 times $1250, or $50, and the premium added to the par value equals $1300, the real value.

OPERATION.

$50 × 25 = $1250 = par value.
.04
50.00 = premium.
1250.
$1300 = real value.

RULE.—*Multiply the par value by the rate, for amount of premium or discount. Add or subtract this from par value, for real value.*

2. I bought 75 shares of gas stock ($20), at 3½% premium; required the premium and the cost.
Ans. $52.50; $1552.50.

3. I sold 120 shares of stock ($15), at 5% discount; required the discount, and the amount received for it.
Ans. $90; $1710.

4. Sold 34 shares of R. R. stock ($50), at 2½% discount; what was received for it? Ans. $1657.50.

5. Bought 12 shares bank stock ($50), at 12½% *premium, and* afterwards sold the same at $60 per share; *did I gain* or lose, and how much? Ans. Gained $45.

UNITED STATES BONDS.

191. ***United States Bonds*** are printed U.S. promises to pay the holder a certain sum of money, with interest payable at stated periods in the interval.

192. U.S. Bonds may be of various kinds:—of these, *Seven-Thirties*, *Five-Twenties*, *Ten-Forties*, and *Twenty-years-Sixes*, are at present the principal. These may also be either *Coupon* or *Registered* bonds.

193. ***Seven-Thirties*** are so called because they draw an annual interest of $7\frac{3}{10}$ per cent., or a daily interest of two cents on every $100. The interest for less than 6 months is always calculated by days. They are indicated thus: 7-30's.

194. ***Five-Twenties*** are so called because they are payable at the option of the Government any time between 5 years and 20 years after they are issued. They are marked 5-20's.

195. ***Ten-Forties*** are so called because they are payable at the option of the Government any time between 10 years and 40 years after they are issued. They are marked 10-40's.

196. ***Seven-Thirties*** as now issued pay $7\frac{3}{10}\%$ interest in *legal currency*. Five-Twenties pay 6% interest in *gold;* and Ten-Forties, 5% interest in *gold*. Interest on each is due semi-annually.

197. ***Coupon Bonds*** have attached to them "coupons," or certificates for each six months' interest accruing, payable as they become due. These coupons may be cut off, and are payable to the bearer of them.

198. ***Registered Bonds*** have no coupons attached, and the interest is payable only to the bond-holder whose name is registered in the Treasury Department, or to his order.

199. United States bonds usually sell at a premium over their face value, and are quoted or priced at their *current value* per $100.

CASE I

200. To find the interest due on a United States Bond.

1. What is the semi-annual interest of a $500 7-30 bond?

SOLUTION.—If on $1 the interest for a year is $7\frac{3}{10}$ cents, on $500 the interest is 500 times $7\frac{3}{10}$, which is $36.50. If the interest for 1 year is $36.50, for $\frac{1}{2}$ year it is $\frac{1}{2}$ of $36.50, or $18.25.

OPERATION.

500
.073
2)36.500
$18.25

2. What is the semi-annual interest on a $500 5-20 bond worth in currency, gold being at a premium of 30%?

SOLUTION.—If the interest for 1 year is 6%, for $\frac{1}{2}$ year it will be 3%. The interest on $500 at 3% is $15.00. Since gold is at a premium, we must calculate the premium on $15 and add it to the $15. 30% of $15 is $4.50, which added to $15 equals $19.50, or worth of the interest in currency.

OPERATION.

500
03
15.00
.30
4.5000
15.
$19.50

RULE.—1. *Find the interest on the face of the bond for the given time and rate.*

2. *In gold-bearing bonds, add the premium on the interest to the interest.*

3. I have $4000 7-30 bonds with the half-year's interest due; how much is due on them? Ans. $146.

4. My father has a $5000 5-20 bond with a half-year's interest due; how much is the interest worth in currency, gold at 35% premium? Ans. $202.50.

5. I hold two $5000 5-20 bonds with the half-year's interest due; how much will I get in "greenbacks," gold at 25% premium? Ans. $375.

6. Miss Smith has $4000 7-30's and $4000 5-20's; for *which does* she get the most interest in currency in a *year,—gold* at 35% premium? Ans $32 on the 5-20's.

Case II.

201. To find the cost of bonds when they are at a premium.

1. What must I pay for a $500 7-30 bond, when 7-30's are at a premium of 5%, or sell at 105?

Solution.—Five per cent. of $500 is $25, which added to $500 equals $525. Hence I must pay $525 for a $500 bond at 5% premium. Or, if a $100 bond is worth $105, my $500 bond is worth five times as much, or $525.

OPERATION.

500
.05
25.00
$525.00
or $105
5
$525

Rule.—1. *Find the premium, and add it to the face of the bond.*

Rule.—2. *Multiply the price per* $100 *by the number of hundreds bought or sold.*

Note.—If there is interest due on the bond, it must be added. If a coupon not due is cut off, deduct the unaccrued interest thus retained.

2. What must I pay for $2000 7-30's, when they command a premium of 4½%, or sell at 104½? Ans. $2090.

3. How much will a $1000 7-30 bond cost, when there is 93 days' interest due and the premium is 4¼%?

Ans. $1066.10.

4. When 7-30's are at a premium of 5½%, how much must I pay for $3500 bonds with 124 days' interest due on them? Ans. $3779.30.

5. I bought $3000 5-20's with 2 months' interest due, the bond being at a premium of 12%; what did it cost me? Ans. $3390.

Note.—No premium for gold is allowed on less than 6 months' interest.

MISCELLANEOUS PROBLEMS.

1. A has $4685, B has $1245 more than A, and C has as much as A and B both; how many dollars has C?

2. C has 438 acres of land, D has 179 acres less, and E has 48 less than C and D together; how many acres have D and E?

3. Henry can walk 30 miles a day, and William can walk 37 miles; how much farther can William walk in 45 days than Henry?

4. Two men start from the same point and walk in opposite directions, one traveling 25, the other 32, miles an hour; how far apart will they be in 148 hours?

5. If a boy can earn $28 a month, and a man $47 a month, how much will 6 boys and 9 men earn in a month?

6. If a clerk earns $150 a month, and spends $48, how much can he save in 12 months, or a year?

7. A merchant gave $8.25 a barrel for 96 barrels of flour, and sold it for $1000; what was the gain?

8. How many bushels of apples can you buy, at $2½ a bushel, for 56 barrels of flour, at $7.50 a barrel?

Ans. 168.

9. A farmer has 137 hens; now, if he should lay out $625 for hens, at the rate of 25 cents apiece, how many would he then have? Ans. 2637.

10. If a steamboat goes 14 miles an hour, and a railroad train 32 miles an hour, how far will the steamboat go while the train goes 672 miles? Ans. 294 miles.

11. Mary and Susan had each 1420 cents; after Mary had given Susan 360 and Susan had given Mary 480, how many had each? Ans. Mary, 1540; Susan, 1300.

12. Six men and 8 boys earned a sum of money, and divided it so that each man had $75 and each boy $63, how much did they earn?

13. A had 369 acres of land, then bought 720 acres,

and then divided the whole into 9 equal farms, and sold 6 of them; how many acres remain?

14. A boy earns $2.50 a day, and pays 75 cents a day for his board; how much can he thus save in a week?

Ans. $9.75.

15. What number must I add to the product of 126 and 72, to make 10000?

16. A lady went to the city with 600 eggs, and sold them at 15 cents a dozen; what did she receive for them?

17. The sum of two numbers is 7809, and one of the numbers is 3725; required the other number, and their difference.

18. The sum of three numbers is 4082; the first number is 1028, the second 2372; required the third number.

19. The difference between two numbers is 709, and one-half of the smaller number equals 482; what is the larger number?

20. A merchant bought 26 bales of cloth, each bale containing 32 pieces, and each piece 24 yards; how many yards did he buy?

21. If a boat sail 378 miles in 9 days, how far can it sail in 15 days at the same rate? Ans. 630 miles.

22. If 28 men can build a lot of wall in 18 days, how many men can build the same wall in 21 days?

Ans. 24 men.

23. A drover bought 365 cows, at $25 a head, and 758 sheep, at $3.50 a head; what was the cost of all?

24. A merchant bought 96 barrels of flour for $960; he sold 58 barrels at $8 a barrel, and the remainder at $12 a barrel; what was the loss?

25. What is the sum of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{8}$?

26. Subtract the sum of $\frac{2}{3}$ and $\frac{3}{4}$ from the sum of $\frac{5}{6}$ and $\frac{7}{8}$.

27. Multiply $\frac{3}{4}$ by $\frac{5}{6}$, and add the result to the product of $\frac{4}{5}$ and $\frac{1}{10}$.

28. Multiply $\frac{6}{8}$ by $\frac{8}{9}$, and divide the result by the product of $\frac{7}{8}$ and $1\frac{2}{3}$.

29. Find the difference between $\frac{1}{7}$ of $\frac{2}{3}$ of $\frac{3}{4}$ and $\frac{4}{5}$ of $\frac{1}{6}$ of $\frac{6}{7}$.

30. If $\frac{2}{3}$ of a barrel of flour costs $5.60, what will 4 barrels cost at the same rate? Ans. $33.60

31. If $\frac{3}{4}$ of a ton of coal is worth $7.50, what will 12 tons cost at the same rate? Ans. $120.

32. A man sold $\frac{3}{5}$ of his land for $9750; what was it all worth at the same rate? Ans. $16250.

33. Mary lost $\frac{1}{4}$ of her money, and then had $960 remaining; how much had she before the loss?

34. If $\frac{2}{5}$ of a ton of coal is worth $4.50, what is $\frac{5}{8}$ of a ton worth at the same rate? Ans. 9.37\frac{1}{2}$.

35. If $\frac{3}{4}$ of a lot of land is worth $234, what is $\frac{5}{8}$ of the same lot worth? Ans. $260.

36. When rye is worth $\frac{3}{4}$ of a dollar a bushel, how many bushels can be bought for $\frac{7}{8}$ of a dollar? Ans. $\frac{7}{8}$.

37. If one yard of cloth is worth $2\frac{1}{2}$ dollars, how many yards can be bought for 5\frac{3}{4}$? Ans. $2\frac{3}{10}$yd.

38. A man owned $\frac{7}{8}$ of a farm, and sold $\frac{1}{3}$ of his share; what part of the whole farm remained? Ans. $\frac{7}{12}$.

39. How many times will 14 gallons fill a vessel that holds $2\frac{1}{3}$ gallons? Ans. 6 times.

40. A father divided 510 acres of land equally among his sons, giving them $63\frac{3}{4}$ acres apiece; how many sons had he?

41. A merchant bought $6\frac{3}{5}$ cords of wood, at 6\frac{2}{3}$ a cord, and paid for it with corn, at $\frac{4}{5}$ of a dollar a bushel; how many bushels did it take? Ans. 55bu.

42. If the sum of two fractions is $\frac{5}{6}$, and one of them is $\frac{2}{7}$, what is the other? Ans. $\frac{23}{42}$.

43. If the difference of two fractions is $\frac{3}{10}$, and the smaller is $\frac{4}{15}$, what is the other fraction? Ans. $\frac{17}{30}$.

44. What fraction multiplied by $2\frac{6}{7}$ will give a product of $1\frac{1}{2}$? Ans. $\frac{21}{40}$.

45. A pole stands $\frac{1}{3}$ in the mud, $\frac{1}{4}$ in the water, and 20 feet out of the water; what is the length of the pole?

Ans. 48 feet.

46. Reduce £12 13s. 10d. to pence.

47. Reduce £17 17s. 9d. 3far. to farthings.

48. Reduce 28lb. 10oz. 16pwt. 22gr. to grains.

49. Reduce 52876 farthings to pounds.

50. Reduce 89726 grains, Troy, to pounds.

51. Reduce 89726 grains, Apothecaries', to pounds.

52. Reduce 31207 drams to higher denominations.

53. How many drams in 1 ton?

54. How many inches in 1 mile?

55. How many square inches in 1 acre?

56. How many square feet in 1 square mile?

57. How many cubic inches in 1 cord of wood?

58. How many pounds of medicine would a physician use in one year, if he averaged 10 prescriptions a day, of 10 grains each? Ans. 6℔ 4℥ 1℈.

59. A merchant bought 6cwt. 32lb. of sugar at $8\frac{1}{2}$ cents a pound; what did it cost?

60. How often will a wheel 18ft. 4in. in circumference revolve in going 50 miles? Ans. 14400 times.

61. How many square rods in a rectangular field 50 rods long and 30 rods wide? Ans. 1500sq. rd.

62. How many acres in a rectangular field 50 rods long and 30 rods wide? Ans. 9A. 1R. 20P.

63. How many rods of fence will enclose the field in the preceding problem? Ans. 160 rods.

64. In an orchard, $\frac{1}{5}$ of the trees bear apples, $\frac{1}{4}$ bear pears, and the remainder, which is 33, bear peaches; how many trees are there in the orchard?

Ans. 60 trees.

65. How many yards of carpeting 1 yard in width will carpet a floor 16 feet long and 9 feet wide?

Ans. 16 yards.

66. How many cubic feet in a pile of wood 25 feet long, 12 feet high, and 4 feet wide?

67. How many cords in a pile of wood 64 feet long, 16 feet high, and 4 feet wide? Ans. 32 cords.

68. If a load of wood be 16 feet long and 6 feet wide, how high must it be to make a cord? Ans. $1\frac{1}{3}$ft.

69. If my bedroom is 12 feet long, 10 feet wide, and 8 feet high, and I breathe 12 cubic feet of air in a minute, how long will it take to breathe as much air as the room holds? Ans. 80min.

70. America was discovered by Columbus, Oct. 11, 1492; how long from that time until Aug. 8, 1865?

71. The Revolution commenced April 19, 1775, and peace was declared January 20, 1783; how long did the war continue?

72. A boy lost $\frac{1}{3}$ of his kite-string, and then added 20 feet, and then found the string was $\frac{4}{5}$ of its original length; what was the length at first? Ans. 240ft.

73. Mary spent 20 per cent. of $500 for a watch, and 20 per cent. of the remainder for a chain; how much then remained? Ans. $320.

74. A man had 250 acres of land, and sold 25% to A, and 20% to B; how much remained?

Ans $137\frac{1}{2}$ acres.

75. A man bought a horse for $250, and sold it at a gain of 20%; what did he receive for it? Ans. $300.

76. A house was bought for $1280, and sold for $1600; what was the gain per cent.? Ans. 25%.

77. Required the interest of $2400 for 6yr. 10mo. 24da., at 6 per cent. Ans. $993.60.

78. Required the amount of $960 for 2yr. 6mo. 12da., at 7 per cent. Ans. $1130.24.

79. I would now like each little boy and girl, who has gone through the book, to tell me how many *months* old, then how many *weeks* old, and then how many *days* old he or she is.

INTRODUCTION

TO THE

METRICAL SYSTEM OF WEIGHTS AND MEASURES.

THE old system of weights and measures in our country is irregular, difficult to learn, and inconvenient to apply. The same is true with the old systems of all nations. Originating by chance, rather than science, they lacked the simplicity of law, and were, therefore, irregular and chaotic.

In 1795, France adopted a system of weights and measures called the Metric System, based upon the decimal method of notation; all the divisions and multiples being by 10. It was regarded as so great an improvement upon the old methods that it has since been adopted by Spain, Belgium, and Portugal, to the exclusion of all other weights and measures, and is in partial use in Holland, Italy, Germany, and Austria, and also in many parts of Spanish America.

In 1864, the British Parliament passed an act permitting its use throughout the empire whenever parties should agree to use it. In 1866, Congress authorized its use in the United States, and provided for its introduction into the post-offices for the weighing of letters and papers.

To facilitate its adoption, a convenient standard of comparison was furnished, by making the new five-cent piece five grams in weight and one fiftieth of a meter, or two centimeters, in diameter. This system will, without doubt, in a few years be in general use in this country.

The *advantages* of the Metric System are numerous and important.

1. It is easily learned; a school-boy can learn it in a single afternoon.

2. It is easily applied, all the operations being the same as in simple numbers.

3. It does away with addition, subtraction, multiplication, division, and reduction of compound numbers and fractions.

4. It will facilitate commerce, giving the nations a universal system of weights and measures.

THE METRIC SYSTEM.

202. The *Metric System* of Weights and Measures is based upon the decimal system of notation.

203. In this system we first establish the unit of each measure, and then multiply and divide it by 10.

204. *Names.*—We first name the *unit* of any measure, and then derive the other denominations by prefixing words to the unit name.

205. The *higher denominations* are expressed by prefixing to the name of the unit,

Deka,	**Hecto,**	**Kilo,**	**Myria.**
10	100	1000	10,000

206. The *lower denominations* are expressed by prefixing to the name of the unit,

Deci,	**Centi,**	**Milli.**
$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

207. *Units.*—The following are the different units, with their English pronunciation:—

Measure.	Unit.	Pronunciation.	Measure.	Unit.	Pronunciation.
LENGTH,	**Meter,**	(meter.)	CAPACITY,	**Liter,**	(leeter.)
SURFACE,	**Are,**	(air.)	WEIGHT,	**Gram,**	(gram.)
VOLUME,	**Stere,**	(stair.)	VALUE,	**Dollar.**	

MEASURE OF LENGTH.

208. The *Meter* is the *unit of length.* It is the ten-millionth part of the distance from the equator to the poles, and equals 39.37 inches, or 3.28 feet.

TABLE.

10 milli**meters** (m.m.)	equal	1 centi**meter,**	c.m.
10 centi**meters**	"	1 deci**meter,**	d.m.
10 deci**meters**	"	1 **meter,**	M.
10 **meters**	"	1 deka**meter,**	D.M.
10 deka**meters**	"	1 hecto**meter,**	H.M.
10 hecto**meters**	"	1 kilo**meter,**	K.M.
*10 kilo***meters**	"	1 myria**meter,**	M.M.

NOTES.—1. The *meter* is very nearly 3 feet, 3 inches, and 3 eighths of an inch in length, which may be easily remembered as the rule of *three threes*.

2. Cloth, etc. are measured by the *meter;* very small distances, by the *millimeter;* great distances, by the *kilometer*.

3. The new 5-cent piece is $\frac{1}{10}$ of a meter in diameter: hence its diameter is $\frac{1}{8}$ of a decimeter, or 2 centimeters.

4. A *decimeter* is about 4 inches; a *kilometer*, about 200 rods, or $\frac{5}{8}$ of a mile; a *millimeter*, about $\frac{1}{25}$ of an inch. The *inch* is about $2\frac{1}{2}$ centimeters; the *foot*, 3 decimeters; the *rod*, 5 meters; the mile, 1600 meters, or 16 hectometers.

QUESTIONS.

1. How many centimeters in a meter?
2. How many millimeters in a meter?
3. How many decimeters in a dekameter?
4. How many meters in a hectometer?
5. How many meters in a kilometer?

MEASURES OF SURFACE.

209. The *Are* is the *unit of surface* used to measure land. The *are* is a *square dekameter*. It equals 119.6 sq. yd., or 0.0247 acre.

TABLE.

10 milli**ares** (m.a.)	equal	1 centi**are,**	c.a.
10 centi**ares**	"	1 deci**are,**	d.a.
10 deci**ares**	"	1 **are,**	A.
10 **ares**	"	1 deka**re,**	D.A.
10 deka**res**	"	1 hecta**re,**	H.A.
10 hecta**res**	"	1 kila**re,**	K.A.
10 kila**res**	"	1 myria**re,**	M.A.

NOTES.—1. The *are*, *centiare*, and *hectare* are the denominations principally used, as these are exact squares. The centiare is a square whose side is 1 meter; the hectare is a square whose side is 100 meters.

The are = 100 square meters. The centiare = 1 square meter.
The hectare = 10,000 square meters.

2. The *deciare* is not a *square*, it is merely the tenth of an are, the *dekare* is not a square, it is merely 10 ares.

8. A *hectare* equals very nearly 2½ acres; a *centiare* equals nearly 1½ sq. yd. An *acre* is very nearly 40 ares.

MEASURES OF OTHER SURFACES.

210. All surfaces besides land are measured by the square meter, square decimeter, etc. The measures are shown by the following table:—

TABLE.

100 sq. millimeters (m.m.²)	= 1 sq. centimeter,	c.m.²
100 sq. centimeters	= 1 sq. decimeter,	d.m.²
100 sq. decimeters	= 1 sq. meter,	M.²

NOTE.—The measures higher than these are not generally used.

QUESTIONS.

1. How many centiares in an are?
2. How many ares in a hectare?
3. How many square meters in an are?
4. How many square decimeters in an are?
5. How many ares in 640 square meters?

MEASURES OF VOLUME.

211. The *Stere* is the *unit of volume.* It is a *cubic meter*, and equals 35.3166 cubic feet, or 1.308 cu. yd.

TABLE.

10 millisteres (m.s.)	equal	1 centistere,	c.s.
10 centisteres	"	1 decistere,	d.s.
10 decisteres	"	1 stere,	S.
10 steres	"	1 dekastere,	D.S.
10 dekasteres	"	1 hectostere,	H.S.
10 hectosteres	"	1 kilostere,	K.S.
10 kilosteres	"	1 myriastere,	M.S.

NOTE.—1. Wood is measured by this measure. The *stere, decistere, and dekastere* are principally used. 3.6 steres, or 36 decisteres, very nearly equal the common cord.

MEASURES OF OTHER VOLUMES.

212. Other solid bodies are usually measured by the *cubic meter* and its divisions. The measures are shown by the following table.

TABLE.

1000 cubic milli**meters** (m.m.^{3})	=1 cubic centi**meter**, c.m.^{3}
1000 cubic centi**meters**	=1 cubic deci**meter**, d.m.^{3}
1000 cubic deci**meters**	=1 cubic **meter**, M.^{3}

NOTE.—The nigher denominations are not generally used.

QUESTIONS.

1. How many centisteres in a stere?
2. How many decisteres in a dekastere?
3. How many dekasteres in a kilostere?
4. How many cubic meters in a hectostere?

MEASURES OF CAPACITY.

213. The ***Liter*** is the *unit of capacity.* It equals a *cubic decimeter;* that is, a cubic vessel whose size is one-tenth of a meter.

This measure is used for measuring liquids and dry substances. The *liter* is a cylinder, and holds 2.1135 pints wine measure, or 1.816 pints dry measure.

TABLE.

10 milli**liters** (m.l.)	equal	1 centi**liter**,	c.l.
10 centi**liters**	"	1 deci**liter**,	d.l.
10 deci**liters**	"	1 **liter**,	L.
10 **liters**	"	1 deka**liter**,	D.L.
10 deka**liters**	"	1 hecto**liter**,	H.L.
10 hecto**liters**	"	1 kilo**liter**,	K.L.
10 kilo**liters**	"	1 myria**liter**,	M.L.

NOTES.—1. The *liter* is principally used in measuring *liquids,* and the *hectoliter* in measuring grains, etc.

2. The *liter* equals nearly $1\frac{1}{16}$ liquid quarts, or $\frac{9}{10}$ of a dry quart, *or nearly* $\frac{1}{35}$ *of a* bushel measure.

3. The *hectoliter* is about $2\frac{5}{8}$ bushels, or $\frac{4}{9}$ of a barrel. 4 liters are a little more than a *gallon;* 35 liters, very nearly a *bushel.*

QUESTIONS.

1. How many liters in a hectoliter?
2. How many liters in a kiloliter?
3. How many deciliters in a dekaliter?
4. How many liters in a cubic meter? Ans. 1000.
5. How many liters in a stere? Ans. 1000.

MEASURE OF WEIGHT.

214. The *Gram* is the *unit of weight.* It is the weight of a cubic centimeter of distilled water at the temperature of melting ice. The gram equals 15.432 troy grains.

TABLE.

10 milli**grams** (m.g.)	equal	1 centi**gram,**	c.g.
10 centi**grams**	"	1 deci**gram,**	d.g.
10 deci**grams**	"	1 **gram,**	G.
10 **grams**	"	1 deka**gram,**	D.G.
10 deka**grams**	"	1 hecto**gram,**	H.G.
10 hecto**grams**	"	1 kilo**gram,**	K.G., or K.
10 kilo**grams**	"	1 myria**gram,**	M.G.

NOTES.—1. The *gram* is used in weighing letters, in mixing and compounding medicines, and in weighing all very light articles. The new 5-cent coin (dated 1866) weighs 5 grams.

2. The *kilogram* is the ordinary unit of weight, and is generally abbreviated into *kilo.* It equals about $2\frac{1}{2}$ pounds avoirdupois. Meat, sugar, etc. are bought and sold by the *kilogram.*

3. In weighing heavy articles, two other weights, the *quintal* (100 kilograms) and the *tonneau* (1000 kilograms), are used. The *tonneau* is between our *short ton* and *long ton.*

4. The *avoirdupois ounce* is about 28 *grams;* the *pound* is a little less than $\frac{1}{2}$ a *kilo.*

QUESTIONS.

1. How many grams in a kilogram?
2. How many milligrams in a gram?
3. How many decigrams in a kilogram?
4. How many hectograms in a myriagram?